Engineering Analysis with Boundary Elements 35 (2011) 1225–1235

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Explicit evaluation of hypersingular boundary integral equations for acoustic sensitivity analysis based on direct differentiation method

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ARTICLE INFO

Article history:
Received 29 October 2010
Accepted 12 May 2011
Available online 1 July 2011

Keywords:
Acoustic shape sensitivity analysis
Direct differentiation method
Boundary element method
Fictitious eigenfrequency
Burton–Miller method
Hypersingularity
Fast multipole method

ABSTRACT

This paper presents a new set of boundary integral equations for three dimensional acoustic shape sensitivity analysis based on the direct differentiation method. A linear combination of the derived equations is used to avoid the fictitious eigenfrequency problem associated with the conventional boundary integral equation method when solving exterior acoustic problems. The strongly singular and hypersingular boundary integrals contained in the equations are evaluated as the Cauchy principal values and Hadamard finite parts for constant element discretization without using any regularization technique in this study. The present boundary integral equations are more efficient to use than the usual ones based on any other singularity subtraction technique and can be applied to the fast multipole boundary element method more readily and efficiently. The effectiveness and accuracy of the present equations are demonstrated through some numerical examples.

1. Introduction

Shape design sensitivity analysis is a procedure to calculate gradients of cost functions defined to obtain the optimum shape of a given structure with respect to shape design variables. The obtained gradients can then be used to determine the direction to search the optimum values of the design variables. Accordingly, acoustic shape sensitivity analysis is usually the first and most important step in acoustic shape design and optimization processes.

Many shape sensitivity analysis methods have been proposed so far based on the finite element method (FEM) [1–3] and the boundary element method (BEM) [4–10]. But because of the highly accurate solutions on the boundary, the reduction of dimensionality, and the incomparable superiority in solving infinite or semi-infinite acoustic field problems, the BEM has been widely applied to acoustic shape sensitivity analyses. In particular, the reduction of dimensionality gives the advantage of easier mesh regenerations to the BEM in shape design or optimization processes.

The BEM based on the conventional boundary integral equation (CBIE) fails, however, to yield unique solutions for exterior acoustic problems at the eigenfrequencies of the associated interior problems [11]. These eigenfrequencies are usually called "fictitious eigenfrequencies" because they do not have any physical significance, but just arise from the drawback of the boundary integral representation when solving exterior acoustic problems. In order to tackle this problem, two main methods, appropriate for practical applications, have been proposed over the last several decades. The Combined Helmholtz Integral Equation Formulation (CHIEF) proposed by Schenck [11] can successfully conquer this problem by adding some additional constraints in the interior domain, which leads to an over-determined system of equations, then solved by a least-square procedure. This method is very simple to implement but determining the suitable number and positions of the interior points may become troublesome as the wave number increases. Thereafter, some modified CHIEF methods have been proposed [12,13], but they do not solve the problem completely, especially, in the high frequency range yet.

A more sound and effective alternative to circumvent the fictitious eigenfrequency problem is the Burton–Miller method which uses a linear combination of the CBIE and its normal derivative (NDBIE) [14] as the boundary integral equation to solve. It has been proved that this combined BIE formula can yield unique solutions for all frequencies if the coupling constant of the two equations is chosen properly [14]. The comparison between the CHIEF approach and the Burton–Miller method can be found in [16]. The advantage of the Burton–Miller method is that there is no need for making difficult decisions of interior points. The main difficulty is the evaluation of the hypersingular boundary integrals involving a double normal derivative of the

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0955-7997/$ - see front matter
doi:10.1016/j.enganabound.2011.05.004

fundamental solution. Burton and Miller introduced a double surface integral method in their original paper [14] to reduce the order of singularity, although such a technique gives rise to numerically tractable kernels, it is very expensive to evaluate a double surface integral. Thereafter, many singularity subtraction techniques have been proposed to enhance the efficiency of evaluating such hypersingular integrals [5–7,17–25]. In the field of acoustic radiation and scattering analysis, for example, Terai [18] evaluated the hypersingular integrals for a flat portion of the boundary when the collocation point is inside the domain but close to the boundary, and then took the limit of the collocation point to the boundary. Chien et al. [19] regularized the hypersingular integrals through some identities related to an interior Laplace problem. Liu et al. [21] developed a weakly singular form of the hypersingular integral equation by employing certain integral identities involving the static Green's function. Yan et al. [24] presented a double surface integral method in which the evaluation of the double surface integral was reduced to the evaluation of two discretized operator matrices. In the field of acoustic sensitivity analysis, for instance, Matsumoto et al. [5] presented a regularization procedure of the singular boundary integral equations by subtracting and adding back the same terms related to the fundamental solution of Laplace's equation. Koo et al. [6] obtained a weakly singular acoustic sensitivity equation by combining the potential equation, then in order to accelerate the computational process, the weakly singular form was further regularized. Arai et al. [7] regularized the hypersingular sensitivity equation by using the fundamental solution of Laplace's equation and some integral identities of it.

Although various singularity subtraction techniques have been proposed to evaluate the hypersingular integrals, most of them are still cumbersome and require extremely complicated numerical procedure in general. Worse is the case when it comes to the fast multipole boundary element method (FMBEM) [26,27], for instance, when the formula is regularized by using the fundamental solution of Laplace's equation, multipole expansion and other translation formulas have to be implemented not only for the fundamental solution and its derivatives of the Helmholtz equation but also for those of Laplace's equation. However, we find that all the hypersingular integrals can be evaluated without any difficulty when constant elements are used to discretize the boundary, and the computational process becomes easier and more efficient than that of any other singularity subtraction techniques and can be extended to the fast BEM approaches such as the FMBEM readily and efficiently.

In this study, a new set of sensitivity boundary integral equations are derived directly from the strongly singular and hypersingular boundary integral equations for constant element discretization. The concept of material derivative [28,29] is employed in deriving these sensitivity equations. In the derivation, we first differentiate the Kirchhoff–Helmholtz integral equation and its normal derivative with respect to an arbitrary design variable, and then take its limit to the boundary to obtain the sensitivity boundary integral equations. As is well known that the evaluation of strongly singular and hypersingular integrals is the crucial part in the derivation. Since constant elements are used to discretize the boundary in this study, such integrals can be evaluated explicitly without any other singularity subtraction technique. The final equations are not only without any singular term but also very easy and efficient to use. With the linear combination of the boundary integral equations derived in this paper, we can implement the FMBEM for exterior acoustic sensitivity analysis more readily and efficiently than with the formulas obtained by any other techniques.

This paper is organized as follows. The BIE formulas for acoustic state analysis are reviewed in Section 2. The new BIE formulas for acoustic sensitivity analysis without singularity are presented in Section 3. In Section 4, the effectiveness and accuracy of the present formulas are demonstrated through some numerical examples. Section 5 concludes the paper with further discussions.

2. Boundary integral equations for acoustic state analysis

The propagation of time-harmonic acoustic waves in a homogeneous and isotropic acoustic medium is described by the well-known Helmholtz equation:

$$\nabla^2 u(x) + k^2 u(x) = 0, \quad x \in \Omega, \tag{1}$$

where ∇^2 is the Laplace operator, $u(x)$ the sound pressure at point x, and $k = 2\pi f/c$ the wave number, f the frequency, c the sound speed.

The boundary conditions on the boundary surface Γ can be written as

$$u(x) = \overline{u}(x) \quad \text{on } \Gamma_u, \tag{2}$$

$$q(x) = \frac{\partial u}{\partial n}(x) = \mathrm{i}\rho\omega\overline{v}(x) \quad \text{on } \Gamma_q, \tag{3}$$

$$u(x) = zv(x) \quad \text{on } \Gamma_z, \tag{4}$$

where $n(x)$ denotes the outward unit normal vector to the boundary Γ at point x, i the imaginary unit, ρ the medium density, ω the circular frequency, $v(x)$ the normal velocity, z the acoustic impedance. The quantities with upper bars are assumed to be known functions prescribed on the boundary. In the case of acoustic radiation, $u(x)$ must also satisfy the Sommerfeld radiation condition, for three dimensional problems, that is

$$\lim_{|x| \to +\infty} |x| \left(\frac{\partial u(x)}{\partial |x|} - \mathrm{i}ku(x) \right) = 0. \tag{5}$$

The integral representation of the solution to the Helmholtz equation is

$$u(x) + \int_\Gamma q^*(x,y)u(y)\,\mathrm{d}\Gamma(y) = \int_\Gamma u^*(x,y)q(y)\,\mathrm{d}\Gamma(y), \quad x \in \Omega, \tag{6}$$

where x is a collocation point, y the source point, and $u^*(x,y)$ the fundamental solution, for three dimensional acoustic problems, given as

$$u^*(x,y) = \frac{\mathrm{e}^{\mathrm{i}kr}}{4\pi r}, \tag{7}$$

where $r = |y-x|$, and $q^*(x,y)$ is the normal derivative of $u^*(x,y)$, given as

$$q^*(x,y) = -\frac{\mathrm{e}^{\mathrm{i}kr}}{4\pi r^2}(1-\mathrm{i}kr)\frac{\partial r}{\partial n(y)}. \tag{8}$$

The derivative of the integral representation (6) in the direction of an arbitrary unit vector $n(x)$ for $x \in \Omega$ is given by

$$q(x) + \int_\Gamma \tilde{q}^*(x,y)u(y)\,\mathrm{d}\Gamma(y) = \int_\Gamma \tilde{u}^*(x,y)q(y)\,\mathrm{d}\Gamma(y), \tag{9}$$

where $(\tilde{\ }) = \partial(\ )/\partial n(x)$, and

$$\tilde{u}^*(x,y) = -\frac{\mathrm{e}^{\mathrm{i}kr}}{4\pi r^2}(1-\mathrm{i}kr)\frac{\partial r}{\partial n(x)}, \tag{10}$$

$$\tilde{q}^*(x,y) = \frac{\mathrm{e}^{\mathrm{i}kr}}{4\pi r^3}\left[(3-3\mathrm{i}kr-k^2r^2)\frac{\partial r}{\partial n(x)}\frac{\partial r}{\partial n(y)} + (1-\mathrm{i}kr)n_i(x)n_i(y)\right], \tag{11}$$

where n_i is the Cartesian component of the unit vector $n(x)$ or $n(y)$. Einstein's summation convention is used throughout the paper, so repeated indices imply a summation over their range. The direction denoted by $n(x)$ is arbitrarily taken when x is