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Advanced sensitivity analysis of the fuzzy assignment problem

Chi-Jen Lin[a,*], Ue-Pyng Wen[b], Pei-Yi Lin[b]

[a] Department of Industrial Engineering and Management, Ta Hwa Institute of Technology, Hsinchu, Taiwan, ROC
[b] Department of Industrial Engineering and Engineering Management, National Tsing Hua University, Hsinchu, Taiwan, ROC

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ABSTRACT

This paper concentrates on sensitivity analysis of the fuzzy assignment problem (FAP). Since most real environments are uncertain, the FAP is more realistic than the assignment problem in application. Owing to the high degeneracy of the FAP, as that of the assignment problem, traditional sensitivity analysis, called Type I sensitivity analysis, which determines the range in which the current optimal basis remains optimal, is impractical. Hence, we attempt to perform other two types of advanced sensitivity analysis, called Type II and Type III sensitivity analysis, to overcome this problem. A labeling algorithm is then presented, where Type II sensitivity analysis is to determine the range of perturbation to keep the current optimal assignment remaining optimal, and Type III sensitivity analysis is to determine the range for which the rate of change of optimal value function remains unchanged. The procedure of the labeling algorithm is divided into two parts: one is when the unassigned cell is perturbed, and the other is when the assigned cell is perturbed. An example is presented to demonstrate that the labeling algorithm is a useful tool for determining the Type II and Type III sensitivity analysis of the FAP.

1. Introduction

Assignment problem (AP) has been widely used in manufacturing and service systems as well as many indirect applications. The AP can be stated as: Let a number n of jobs be given that must be performed by n workers, where the costs depend on the specific assignments. Each job must be assigned to one and only one worker and each worker has to perform one and only one job. The problem is to find such an assignment that the total cost becomes a minimum [1]. An AP can be formulated as a 0–1 integer programming or linear programming problem as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = 1 \quad \text{for} \quad i = 1, 2, \ldots, n, \\ & \sum_{i=1}^{n} x_{ij} = 1 \quad \text{for} \quad j = 1, 2, \ldots, n, \\ & x_{ij} \in \{0, 1\} \quad \text{or} \quad x_{ij} \geq 0 \quad \text{for} \quad i, j = 1, 2, \ldots, n; \end{aligned} \tag{1.1}$$

where c_{ij}, the only input parameter, represents the cost associated with worker i (i = 1,2,. . .,n) who has performed job j (j = 1,2,. . .,n). All the c_{ij}'s are deterministic numbers and usually denoted as an $n \times n$ matrix $\mathbf{C} = [c_{ij}]$. An assignment problem is then defined to choose precisely one element from each row and column so that the total sum is minimized.

However, in many real-world applications, costs are not deterministic numbers. In recent years, many researchers have begun to investigate assignment problem and its variants under fuzzy environments. Belacela and Boulasselb [2] studied a multi-criteria fuzzy assignment method applied to medical diagnosis. Ridwan [3] studied a fuzzy preference-based traffic assignment problem. Liu and Li [4] studied the fuzzy quadratic assignment problem with penalty. Feng and Yang [5] studied a two-objective fuzzy k-cardinality assignment problem. Liu and Gao [6] studied the fuzzy weighted equilibrium multi-job assignment problem.

Lin and Wen [7] investigated a fuzzy assignment problem (FAP) in which the cost depends on the quality of the job. The quality of job is regarded as the performance of worker. The manager of the workers also has to limit the total cost with a range, and thus the total cost is related to the performance of the manager. Hence, they define the membership function of cost, denoted as $\tilde{c}_{ij}$, as the linear monotone increasing function shown in (1.2). α_{ij} is the minimum cost for worker i to perform job j, and β_{ij} is the minimum cost associated with worker i to perform job j at the highest quality q_{ij}. The greater the cost spent, between α_{ij} and β_{ij}, the higher the quality is attained. However, any expense exceeding β_{ij} is useless since the quality can no longer be enhanced. Without loss of generality, it is assumed that $0 < \alpha_{ij} < \beta_{ij}$ and they define the quality matrix $[q_{ij}]$ where $0 < q_{ij} \leq 1$. Condition $x_{ij} = 1$ is added to (1.2) because there is

* Corresponding author.
E-mail address: lcj@thit.edu.tw (C.-J. Lin).

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no real expense if $x_{ij}=0$ in any feasible solution.

$$\mu_{ij}(c_{ij})=\begin{cases} q_{ij} & \text{if} \quad c_{ij}\ge\beta_{ij}, \quad x_{ij}=1,\\ \dfrac{q_{ij}(c_{ij}-\alpha_{ij})}{\beta_{ij}-\alpha_{ij}} & \text{if} \quad \alpha_{ij}\le c_{ij}\le\beta_{ij}, \quad x_{ij}=1,\\ 0 & \text{otherwise.}\end{cases} \tag{1.2}$$

Moreover, notation $\langle\alpha_{ij},\beta_{ij}\rangle$ is employed to denote $\tilde{c}_{ij}$. Matrix $[\tilde{c}_{ij}]$ is shown as follows:

$$[\tilde{c}_{ij}]=[\langle\alpha_{ij},\beta_{ij}\rangle]$$

In addition, all the α_{ij}'s form the matrix $[\alpha_{ij}]$ and all the β_{ij}'s form the matrix $[\beta_{ij}]$.

They also define the membership function of total cost $\tilde{c}_T$, which is related to the performance of the manager, as the linear monotonically decreasing function in (1.3). Numbers a and b are the lower and upper bounds of total cost, respectively; and notation $\langle a, b\rangle$ denotes the fuzzy interval $\tilde{c}_T$. It is suggested that a number less than or equal to the minimum assignment of matrix $[\alpha_{ij}]$ should be taken as a, and a number larger than or equal to the maximum assignment of matrix $[\beta_{ij}]$ should be taken as b.

$$\begin{aligned}\mu_T(c_T)&=\mu_T\left(\sum_{i=1}^{n}\sum_{j=1}^{n}c_{ij}x_{ij}\right)\\ &=\begin{cases}1, & c_T\le a,\\ \dfrac{b-\sum_{i=1}^{n}\sum_{j=1}^{n}c_{ij}x_{ij}}{b-a}=\dfrac{b-c_T}{b-a}, & a\le c_T\le b,\\ 0, & c_T\ge b.\end{cases}\end{aligned} \tag{1.3}$$

Assume that a team comprises all workers and the manager. The performance of the team is determined by taking the lowest performance of members in the group, the company has to equally emphasize the performance of each member in the group. Hence, the Bellman–Zadeh's criterion [8] is used and the FAP can be derived as:

$$\begin{aligned}\max\quad & \frac{b-\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ij}x_{ij}}{b-a+\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}x_{ij}}\\ \text{s.t.}\quad & \sum_{j=1}^{n}x_{ij}=1 \quad \text{for} \quad i=1,2,\ldots,n\\ & \sum_{j=1}^{n}x_{ij}=1 \quad \text{for} \quad j=1,2,\ldots,n\\ & x_{ij}\in\{0,1\}\ \text{or}\ x_{ij}\ge 0 \quad \text{for} \quad i,j=1,2,\ldots,n\end{aligned} \tag{1.4}$$

where $\gamma_{ij}=(\beta_{ij}-\alpha_{ij})/q_{ij}$ for $\forall\, i, j$, is the slope of the membership function.

Furthermore, Lin and Wen [7] proposed an efficient labeling algorithm, denoted as the LW algorithm, for solving (1.4), which is a linear fractional programming problem and extreme-point optimum exists. Note that the constraints of (1.4) are identical to those of (1.1). Hence, the optimal solution of (1.4) is inherently highly primal degenerate and corresponds to several different optimal bases. Consequently, changing the optimal basis does not ensure that the optimal assignment will be changed. Degeneracy thus makes the traditional sensitivity analysis, an important issue after obtaining the optimal solution, impractical.

The main objective of this paper is to investigate the properties of the FAP and then propose a modified labeling algorithm to determine two other types of practical sensitivity range. The rest of this paper is organized as follows. Section 2 reviews the concepts of the LW algorithm, types of sensitivity range and various perturbations of the FAP. We then propose the algorithm which is extended from the LW algorithm to determine the two types of sensitivity range in Sections 3 and 4, respectively. Furthermore, two numerical examples are presented in order to demonstrate the proposed approaches. Section 5 concludes the study with a brief summary.

Table 1
Parameters for cells of LW algorithm.

If $x_{ij}=1$, i.e., an assigned cell	If $x_{ij}=0$, i.e., an unassigned cell
α_{ij} (top left), ① (centre), γ_{ij} (bottom right)	α_{ij} (top left), Q_{ij} (centre), γ_{ij} (bottom right)

2. LW algorithm and sensitivity analysis

2.1. Concepts of the LW algorithm

$$\text{Let} \quad \rho=\frac{1}{b-a+\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}x_{ij}}>0 \tag{2.1}$$

$$\text{and} \quad y_{ij}=\rho\cdot x_{ij} \quad \text{for} \quad i,j=1,2,\ldots,n \tag{2.2}$$

(1.4) can then be converted into a linear programming model as follows:

$$\begin{aligned}\max\quad & -\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ij}y_{ij}+b\cdot\rho\\ \text{s.t.}\quad & \sum_{j=1}^{n}y_{ij}-\rho=0 \quad \text{for} \quad i=1,2,\ldots,n\\ & \sum_{i=1}^{n}y_{ij}-\rho=0 \quad \text{for} \quad j=1,2,\ldots,n\\ & \sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}y_{ij}+(b-a)\rho=1\\ & y_{ij},\rho\ge 0 \qquad \text{for} \quad i,j=1,2,\ldots,n\end{aligned} \tag{2.3}$$

Suppose $u_1,\ldots,u_n$, $v_1,\ldots,v_n$ and f are the dual variables of this model, then its corresponding dual problem is as follows:

$$\begin{aligned}\min\quad & f\\ \text{s.t.}\quad & u_i+v_j+\gamma_{ij}f\ge-\alpha_{ij} \quad \text{for} \quad i,j=1,2,\ldots,n\\ & -\sum_{i=1}^{n}u_i-\sum_{j=1}^{n}v_j+(b-a)f\ge b\\ & u_1,\ldots,u_n, \quad v_1,\ldots,v_n, \quad f\in R\end{aligned} \tag{2.4}$$

The LW algorithm begins with a feasible solution of (2.3), i.e., of (1.4), and proceeds to obtain a feasible solution of (2.4) while maintaining complementary slackness. At each stage of the solution procedure, a better solution of (2.3) is obtained until the optimal solution is reached. The steps of the LW algorithm are performed directly on a table as that of classical transportation simplex method. Parameters corresponding to cell (i, j) are displayed in Table 1. Lin and Wen [7] demonstrated that the LW algorithm offers an effective and efficient method for solving FAP.

2.2. Three types of sensitivity range

Koltai and Terlaky [9] showed that managerial questions are not answered satisfactorily with the mathematical interpretation of the traditional sensitivity analysis when the solution of a linear programming model is degenerate. In order to perform proper sensitivity analysis for (1.4), two other types of sensitivity analysis must be considered depending on how the analysis is performed. Here, we summarize three types of sensitivity analysis as follows: [9–11]