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A wideband fast multipole boundary element method for three dimensional acoustic shape sensitivity analysis based on direct differentiation method

Changjun Zheng [a], Toshiro Matsumoto [b,*], Toru Takahashi [b], Haibo Chen [a]

[a] Department of Modern Mechanics, University of Science and Technology of China, Hefei, Anhui 230027, PR China
[b] Department of Mechanical Science and Engineering, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8604, Japan

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ABSTRACT

This paper presents a wideband fast multipole boundary element approach for three dimensional acoustic shape sensitivity analysis. The Burton–Miller method is adopted to tackle the fictitious eigenfrequency problem associated with the conventional boundary integral equation method in solving exterior acoustic wave problems. The sensitivity boundary integral equations are obtained by the direct differentiation method, and the concept of material derivative is used in the derivation. The iterative solver generalized minimal residual method (GMRES) and the wideband fast multipole method are employed to improve the overall computational efficiency. Several numerical examples are given to demonstrate the accuracy and efficiency of the present method.

1. Introduction

Acoustic shape sensitivity analysis is an important step of acoustic design and optimization processes. It can provide information on how the geometry change affects the acoustic performance of the given structure. So far, many sensitivity analysis methods have been proposed based on the finite element method (FEM) [1–3] and the boundary element method (BEM) [4–10]. But due to the convenience in remeshing compared with the FEM as well as relatively good accuracy of the solutions on the boundary and especially the incomparable superiority in solving infinite or semi-infinite acoustic problems, the BEM has been very widely used in acoustic sensitivity analyses [11,12]. Matsumoto et al. [5] and Koo et al. [6] derived two different acoustic sensitivity boundary integral equations with respect to shape design variables. Kim and Dong [7] presented a shape design sensitivity formulation for structural-acoustic problems using sequential finite element and boundary element methods. However, in these researches, the authors started from the conventional boundary integral equation (CBIE) but did not consider the fictitious eigenfrequency problem of it in solving exterior acoustic wave problems. Smith and Bernhard [8] proposed a semi-analytical sensitivity formulation and resolved the fictitious eigenfrequency problem by using the CHIEF method [13]. Arai et al. [9] derived an analytical shape sensitivity equation based on a modified Burton–Miller formulation [14].

However, though the BEM reduces the problem dimensionality by one, the conventional boundary element method (CBEM) produces a fully populated, non-symmetric and sometimes ill-conditioned coefficient matrix which leads to increased storage requirements and machine time in comparison with domain methods such as the FEM and the finite difference method (FDM). This well-known drawback had constrained the extensive usage of the BEM in large-scale engineering applications for several decades. For instance, as for a problem involving N degrees of freedom, a direct solver, such as the Gauss elimination method requires the storage of $O(N^2)$ and the solution cost of $O(N^3)$. Even worse in the shape sensitivity analysis by the direct differentiation method, more integral evaluations for each pair of boundary elements are needed. While, use of iterative solvers, such as the GMRES [15], does not reduce the storage requirements but can reduce the solution cost to $O(MN^2)$, where M is the number of iterations required, and $O(N^2)$ per iteration cost arising from the dense matrix–vector product. This is still quite expensive for large-scale problems. In order to further improve the efficiency and reduce the storage requirements of the BEM with iterative solvers, various acceleration techniques, such as the fast multipole method (FMM) [16], the fast wavelet transforms [17], the precorrected-FFT [18], and the H-matrices [19], have been proposed to accelerate the matrix–vector product. Among these methods, the FMM seems to be one of the most widely accepted methods in the fast BEM

* Corresponding author. Tel.: +81 52 789 2780; fax: +81 52 789 3123.
E-mail addresses: cjzheng@mail.ustc.edu.cn (C. Zheng), t.matsumoto@nuem.nagoya-u.ac.jp (T. Matsumoto), ttaka@nuem.nagoya-u.ac.jp (T. Takahashi), hbchen@ustc.edu.cn (H. Chen).

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community. Moreover, it has also been acclaimed as one of the top 10 algorithms of the 20th century [20]. The FMM allows the matrix–vector product to be performed to a given precision in $O(N)$ operations and reduces the storage requirements to $O(N)$ as well, for instance, for potential problems or low-frequency acoustic wave problems. This method was first introduced by Greengard and Rokhlin [16], and then intensively studied and extended to the solution of problems arising from the Laplace, Helmholtz, Maxwell, and other equations [21–38]. A comprehensive review can be found in [39].

This paper promotes the applications of fast multipole boundary element method (FMBEM) in three dimensional acoustic shape sensitivity analyses. In order to tackle the fictitious eigenfrequency problem when solving exterior acoustic problems, the Burton–Miller method [40] is employed in this study. The Burton-Miller method is a linear combination of the CBIE and the normal derivative boundary integral equation (NDBIE), and has been proved to yield unique solutions for all frequencies if the coupling constant of the two equations is chosen properly [40,41]. However, the most difficult part in implementing this approach is that the NDBIE is a hypersingular type involving a double normal derivative of the fundamental solution. Although various singularity subtration techniques have been proposed to evaluate such hypersingular terms, most of them are still cumbersome and require extremely complicated numerical procedure in general [5,6,9,42–48]. Worse is the case when it comes to the FMBEM, for instance when the formulation is regularized by using the fundamental solution of Laplace's equation, multipole expansion formulations and other translation formulations have to be implemented not only for the fundamental solution and its derivatives of the Helmholtz equation but also for those of Laplace's equation. However, the constant element is used to discretize the problem boundary in this study, and in this case all the hypersingular boundary integrals can be evaluated explicitly and directly. Hence the computational process is more efficient than that of any other singularity subtraction technique [49–51].

This paper is organized as follows. The BEM and FMM formulations for the boundary integral equations for the sensitivity coefficients derived based on the direct differentiation method are introduced in Section 2. The wideband FMM algorithm is presented in Section 3. Section 4 gives three numerical examples to demonstrate the validity and efficiency of the method. Section 5 concludes the paper with further discussions.

2. Formulations

2.1. BEM formulations

The governing differential equation for the propagation of time-harmonic acoustic waves in a homogeneous and isotropic acoustic medium is the following Helmholtz equation:

$$\nabla^2 u(x) + k^2 u(x) = 0, \tag{1}$$

where ∇^2 is the Laplace operator, $u(x)$ the sound pressure at the field point x, and $k=\omega/c$ the wave number, ω the angular frequency of the acoustic wave, c the sound speed. In Eq. (1), a steady-state excitation with a time factor $e^{-i\omega t}$ is assumed, where i is the imaginary unit and t the time. Since $u(x)$ is the amplitude of the sound pressure and its phase changes from ωt, its value is assumed as a complex number.

The boundary conditions are given as

$$u(x) = \bar{u}(x) \quad \text{on } \Gamma_u, \tag{2}$$

$$q(x) = \frac{\partial u}{\partial n}(x) = i\rho\omega\bar{v}(x) \quad \text{on } \Gamma_q, \tag{3}$$

$$u(x) = zv(x) \quad \text{on } \Gamma_z, \tag{4}$$

where $n(x)$ denotes the unit outward normal vector at point x, ρ the density of the medium, $v(x)$ the normal velocity, z the acoustic impedance. The quantities with overbars indicate given values on the boundary. As for exterior acoustic wave problems, it is necessary to introduce a condition at infinity which ensures that all scattered and radiated waves are outgoing. This is usually referred to as the Sommerfeld radiation condition [52], for three dimensional case, it is expressed as

$$\lim_{|x|\to+\infty} |x| \left(\frac{\partial u(x)}{\partial |x|} - iku(x) \right) = 0. \tag{5}$$

2.1.1. BEM formulations for acoustic state analysis

The integral representation of the solution to the Helmholtz equation is

$$u(x) + \int_\Gamma q^*(x,y)u(y)\,d\Gamma(y) = \int_\Gamma u^*(x,y)q(y)\,d\Gamma(y), \tag{6}$$

where y is the source point on the boundary surface Γ, and $u^*(x,y)$ the fundamental solution, for three dimensional acoustic wave problems, given as

$$u^*(x,y) = \frac{e^{ikr}}{4\pi r}, \quad \text{with } r = |y-x|, \tag{7}$$

and $q^*(x,y)$ is the normal derivative of $u^*(x,y)$, i.e.,

$$q^*(x,y) = \frac{\partial u^*(x,y)}{\partial n(y)} = -\frac{e^{ikr}}{4\pi r^2}(1-ikr)\frac{\partial r}{\partial n(y)}. \tag{8}$$

Taking the limit of point x to the boundary Γ in Eq. (6) leads to the following CBIE:

$$C(x)u(x) + \fint_\Gamma q^*(x,y)u(y)\,d\Gamma(y) = \int_\Gamma u^*(x,y)q(y)\,d\Gamma(y), \tag{9}$$

where the coefficient $C(x)$ is 1/2 if Γ is smooth around x, and the symbol $\fint$ denotes that the integral is evaluated in the sense of Cauchy principal value. It is well known that the CBIE cannot yield unique solutions for exterior acoustic wave problems at the eigenfrequencies of the corresponding interior problems [40]. These eigenfrequencies are referred to as the *fictitious eigenfrequencies* because they have no physical meaning. A remedy for this problem is to combine the NDBIE with the CBIE linearly together [40].

The directional derivative of the integral representation (6) in the direction $n(x)$ is given as

$$q(x) + \int_\Gamma \tilde{q}^*(x,y)u(y)\,d\Gamma(y) = \int_\Gamma \tilde{u}^*(x,y)q(y)\,d\Gamma(y), \tag{10}$$

where $\tilde{()} = \partial()/\partial n(x)$ and

$$\tilde{u}^*(x,y) = -\frac{e^{ikr}}{4\pi r^2}(1-ikr)\frac{\partial r}{\partial n(x)}, \tag{11}$$

$$\tilde{q}^*(x,y) = \frac{e^{ikr}}{4\pi r^3}\left[(3-3ikr-k^2r^2)\frac{\partial r}{\partial n(x)}\frac{\partial r}{\partial n(y)} + (1-ikr)n_i(x)n_i(y)\right]. \tag{12}$$

In Eq. (12), n_i is the cartesian component of the vector $n(x)$ or $n(y)$. Einstein's summation convention applies throughout this paper, so repeated indices imply summation over their range.

Letting point x approach the boundary Γ in Eq. (10) gives the following NDBIE:

$$C(x)q(x) + \fint_\Gamma \tilde{q}^*(x,y)u(y)\,d\Gamma(y) = \fint_\Gamma \tilde{u}^*(x,y)q(y)\,d\Gamma(y), \tag{13}$$

where the symbol $\fint$ indicates that the integration is carried out in the sense of Hadamard finite part of the divergent integral.