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Sensitivity analysis of flexible joint nonholonomic wheeled mobile manipulator in singular configuration

A.H. Korayem *, V. Azimirad, H. Binabaji, M.H. Korayem

Robotic Research Laboratory, Center of Excellence in Experimental Solid Mechanics and Dynamics, School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran

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ABSTRACT

This paper proposes a method for decreasing jerk and increasing Maximum Allowable Load (MAL) of nonholonomic Wheeled Mobile Manipulator (WMM) considering flexibility of joints in singular conditions. The full dynamic model of nonholonomic WMM contains simultaneous operation of mobile base and manipulator with joint flexibility (in wheels and manipulator) which is presented here. The problem is formulated in terms of the optimal control which leads to a two point boundary value problem. Then Sobol's sensitivity analysis method is applied to determine the optimal values of flexible joint constants subject to the jerk minimization. To illustrate the proposed method, two categories of conditions are considered: conditions containing non-singular configuration and the singular conditions. An example is explained for non-singular condition of nonholonomic WMM in presence of obstacle in which a complex path is generated but there is no singularity in robot configuration. Some examples of occurring singular configuration in final point and moving boundary condition is also presented. The results show that flexibility of the joints near to singular configuration normalizes the sudden movement and jerk implied to actuators. That is why using a rotational spring with a low stiffness coefficient could be helpful to decrease the high jerk and increase the maximum allowable load in mobile robots.

1. Introduction

Carrying the maximum allowable load according to bounded energy and actuator limitation in terms of torque and jerk in mobile manipulators is a vital factor, but nearing to singular condition must be avoided because of imposing high amounts of jerk to actuators. On the other hand, the robot performance changes in flexible joint robots. In addition, this property causes some differences in determination of maximum allowable load compared to a rigid model. For example, considering flexibility in joints will cause vibration, especially on fast turnings and in an environment with an obstacle, in which to prevent colliding, robot must move in complex paths. Determining maximum allowable load for a nonholonomic mobile robot with flexible joints in manipulator and wheels of the mobile base has applications in advanced trajectory planning, design and identification of motors size. Previous works in this area mostly considered the accuracy and deviation of the end effector [1].

The approaches used to solve the open loop optimal control problems are classified in direct and indirect methods. Direct methods are based on the conversion of the optimal control problem into a parameter optimization problem [2]. In a series of papers, some research effort has been paid to establish the maximum load carrying capacity of linear moving of rigid base holonomic manipulators by

* Corresponding author. Tel.: +98 21 77240194;
fax: +98 21 77240488.
E-mail address: hkorayem@iust.ac.ir (A.H. Korayem).

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Nomenclature

O	center of the world coordination system
o	intersection of the axis of symmetry with the driving wheel axis
b	distance between driving wheels and axis of symmetry
C	center of mass of mobile platform
r	radius of each wheel
d	distance from c to o
l_1	length of first link
l_2	length of second link
(x, y)	coordination of center of mass C in the world coordination system
(X_e, Y_e)	coordination of end effector in the world coordination system
θ_r	angular displacement of right wheel
θ_l	angular displacement of left wheel
θ_1	angular displacement of first link from the mobile platform
θ_2	angular displacement of second link from first link
τ_1	torque exerted to right wheel
τ_2	torque exerted to left wheel
τ_3	torque exerted to first joint
τ_4	torque exerted to second joint
ϕ	heading angle of platform measured from X-axis of the world coordinates
k	4×4 diagonal matrix of stiffness coefficients of joints
m_p	mass of payload
$M(q_s)$	$s\times s$ inertial matrix
$V(q_s,\dot{q}_s)$	$s\times1$ non-linear terms matrix
$E(q)$	fix matrix
λ	Lagrange multipliers
t_f	final time
R	weighting matrix for velocity of joints and wheels in cost function
Q	weighting matrix for torque exerted to joints and wheels in cost function
w_{jk}^i	weighting matrix related to obstacle avoidance of parts in cost function
I_r	moment of inertia of motor and gears
D	variance
S	sensitivity index

direct methods. Although the kinematic analysis of WMM has been considered a lot but the dynamic analysis is few [1]. Recently, MAL for a holonomic flexible arm mounted on a track with linear moving is dealt with, [3] but flexibility in wheels' joints was not considered. In addition, it has to be mentioned that the word "mobile" was referred to a linearly moving holonomic track. Important parameters which must be considered in path planning problem are torque and jerk, but none of the previous works considered the effect of joint flexibility on jerk.

Although torque consideration in the most previous work has been done, but jerk limitation which has a considerable effect on deviation from desired path [4] and decreasing the actuator lifetime, is not considered in determination of MAL. Engelbrecht studied the minimum principles in path planning like the jerk but for complex systems such as WMM, it results in very large volume of equations [5]. Macfarlane and Croft proposed an algorithm for finding jerk bounded trajectories [4]. Mattmüller and Gisler determined smooth and near to time-optimal path-constrained trajectories, in which not only the velocity and the acceleration but also the jerk are explicitly constrained [6]. Beside, Zhu et al. proposed time-optimal and jerk-continuous trajectory planning algorithm to provide ideal trajectories for joint controller with the minimum traveling time [7]. Ider proposed a jerk bounded path equation for flexible joint fix robots but they did not study the WMMs and singular condition [8].

On the other hand, some researchers have focused on singularity for rigid joint WMM. Bayle discussed the manipulability ellipsoid for a rigid mobile manipulator regarding to internal configuration of mobile base [9]. Kim et al. studied the configuration dependent singularity in WMMs in geometrical viewpoint [10]. Korayem et al. described a path planning technique for obtaining the MAL of rigid joint nonholonomic wheeled mobile manipulator in presence of obstacles [11]. However no work has been reported on the effect of joint-flexibility on jerk and MAL in singular configuration.

The subject of this paper is analysis of jerk and the MAL determination of nonholonomic mobile manipulator with joint flexibility in different environmental condition and configuration including the presence of obstacle in which WMM must track a complex path but there is no singularity in the configuration, obstacle free motion with singular configuration of the manipulator and moving end boundary condition in which optimality results in minimum motion of mobile base, extended configuration of manipulator and nearing to the singularity. The problem is stated as an optimal control problem and applying iterative algorithm to increase the payload, according to physical constraints and actuator limitations (torque and jerk). This paper is structured as follows. The configuration of the mobile manipulator with flexible joints (in wheels and manipulator) and its full dynamic model based on coordinated motion of vehicle and manipulator are described in Section 2. In Section 3, the algorithm and Sobol's sensitivity analysis method are introduced. In Section 4 the sensitivity analysis of jerk and following simulation results for various conditions of flexible and rigid joint (in both non-singular condition and singular condition) is described, Section 5 deals with the effect of joint flexibility on jerk and MAL in moving boundary problems. Finally the discussion is done for decreasing the jerk.

2. Wheeled mobile manipulator formulation

The rigid and flexible joint WMM must be modeled to show the effect of joint flexibility on jerk and MAL.