Mechanical Systems and Signal Processing 28 (2012) 482–491

Contents lists available at SciVerse ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

# Parametric sensitivity analysis of the importance measure

Lijie Cui, Zhenzhou Lu *, Qi Wang

*School of Aeronautics, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, PR China*

A R T I C L E  I N F O

*Article history:*
Received 26 April 2011
Received in revised form
22 July 2011
Accepted 11 October 2011
Available online 25 November 2011

*Keywords:*
Parametric sensitivity analysis
Importance measure
Moment-independent
Reliability
Probability density function evolution method

A B S T R A C T

To overcome the difficulties in computing the parametric sensitivity of the importance measure, a new moment-independent importance measure based on the cumulative distribution function is proposed to represent the effects of model inputs on the uncertainty of the output. Based on that, definitions of the parametric sensitivities of the importance measure are given, and their computational formulae are derived. The parametric sensitivities illustrate the influences of varying some variables' distribution parameters to the input variables' importance measures, which provide an important reference to improve or change the performance properties. The probability density function evolution method, an efficient tool due to its high efficiency and precision, is applied into computing the proposed importance measure and its parametric sensitivities. Finally, three examples including the Ishigami test function, a structure model and a mechanism model are adopted to illustrate the feasibility and correctness of the proposed indices and solution.



## 1. Introduction

Sensitivity analysis (SA) has been widely used in engineering design to gain more knowledge of complex model behavior and help designers make informed decisions regarding where to spend the engineering effort [1]. Saltelli et al. [2] classified the sensitivity analysis techniques into two groups: local SA methods and global SA methods. The local SA techniques are the set of methods that find the rate of change in the model output by varying input variables one at a time near a given central point. For design under uncertainty, sensitivity analysis is performed with respect to the probabilistic characteristics of model inputs and outputs. The term global SA is used when the focus is on studying the impact of variations over the entire range of model inputs – as opposed to local SA using partial derivatives – on the variation of a model output [3]. The family of global SA indicators principally include non-parametric techniques suggested by Storlie et al. [4–7] and Helton et al. [8], variance-based methods suggested by Sobol [9–11], and further developed by Iman et al. [12,13] and Homma et al. [14], moment-independent techniques, respectively, proposed by Chun et al. [15], Liu et al. [16,17], Borgonvo [18] and Cui et al. [19]. Indicators created for global SA are called global importance measure or uncertainty importance measure (IM). The moment-independent IM overcomes the disadvantages of other global SA methods, and provides more complete and proper information to reflect the effects of uncertain input variables on the uncertain output response so that they are the most popular global SA techniques.

The global SA can be used in the prior-design stage for variables screening when a design solution is yet identified and the post-design stage for uncertainty reduction after an optimal design has been determined [17]. In engineering, most of the uncertain inputs are assumed as random variables obeying certain distributions. Obviously, the uncertainty

* Corresponding author. Tel./fax: +86 29 88460480.
*E-mail addresses:* cuilijie@mail.nwpu.edu.cn (L. Cui), zhenzhoulu@nwpu.edu.cn (Z. Lu), wangqitop@hotmail.com (Q. Wang).

0888-3270/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ymssp.2011.10.015

of model input is decided by its distribution parameters. If one can combine the global SA of input variable with the local SA of input parameters, the effects of changing the distribution parameters on the IMs of variables can be known, and one can directly change the input's IMs by controlling or modifying some input's distribution parameters, namely changing the input's distribution parameters can also influence the uncertainty of the output, which would facilitate its use under various scenarios of design under uncertainty, for instance in robust design, reliability-based design, and utility optimization.

As stated above, the moment-independent IM presented in [18] can properly and completely reflect the effect of the uncertain input on the output. However, there are two aspects of difficulties in analyzing the parametric sensitivity of IM. On one side, the absolute operation existing in computing the IM cannot compute its derivative in most cases. On the other side, it is very difficult to compute the derivative of the PDF of the output response without a definite analytical expression, because the analytical expression is difficult to obtain in most engineering problems. To keep the physical meaning of the existed moment-independent IM unchanged and make the parametric sensitivity of the IM compute feasible, a new moment-independent IM is proposed to represent the effect of the input variable on the entire distribution of the output uncertainty. Furthermore, solution for computing the proposed IM and its parametric sensitivity is established based on the probability density function evolution method (PDEM) to avoid expensive computational cost.

## 2. The parametric SA of the IM

### 2.1. Difficulties in solving the parametric sensitivity of the IM proposed by Borgonovo

Given a performance response function $Y = g(X_1, X_2, \ldots, X_n)$, where $Y$ is the model output, and $(X_1, X_2, \ldots, X_n)$ are the input random variables. Denote $f_Y(y)$ and $f_{Y|X_i}(y)$ as the unconditional and conditional probability density function (PDF), respectively, $f_Y(y)$ is obtained with all input variables varying over their variation range, and $f_{Y|X_i}(y)$ is obtained by assuming input variable $X_i$ at a fixed value $x_i$. The absolute value of the difference between $f_Y(y)$ and $f_{Y|X_i}(y)$ can represent the effect of $X_i$ on the distribution of $Y$ when $X_i$ is $x_i$ (Fig. 1). When $X_i$ takes values in its distributed field, the expected effect of $X_i$ on the distribution of $Y$ can be measured by the moment-independent IM $\delta_i$ [18]

$$\delta_i = \frac{1}{2}\int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty}|f_Y(y) - f_{Y|X_i}(y)|\,\mathrm{d}y\} f_{X_i}(x_i)\mathrm{d}x_i\right\} \quad (1)$$

The following problems exist in analyzing the parametric sensitivity of the moment-independent IM $\delta_i$.

(1) The derivative of IM $\delta_i$ to the distribution parameters must be available when analyzing the parametric sensitivity of IM, but there is an absolute operation in Eq. (1), which causes many difficulties in the computational process.
(2) In computing the parametric sensitivity of IM $\delta_i$, for example, the sensitivity of $\delta_i$ to $\theta_{X_{ij}}$, $\theta_{X_{ij}}$ is the $j$th distributional parameter of input variable $X_i$, $\partial f_Y(y)/\partial\theta_{X_{ij}}$ must be calculated. Because there is not a definite expression of the output response $Y$ in most engineering applications so that the expression of $f_Y(y)$ cannot be obtained, and $\partial f_Y(y)/\partial\theta_{X_{ij}}$ only can

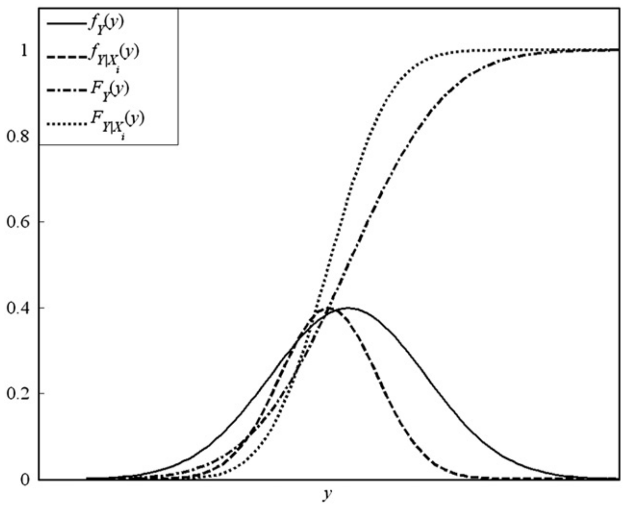


**Fig. 1**