Mechanical Systems and Signal Processing 29 (2012) 213–227

Contents lists available at SciVerse ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

On the identification of hysteretic systems. Part II: Bayesian sensitivity analysis and parameter confidence

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ARTICLE INFO

Article history:
Received 31 May 2011
Accepted 10 January 2012
Available online 7 February 2012

Keywords:
Hysteresis
The Bouc–Wen model
Nonlinear system identification
Bayesian sensitivity analysis
Parameter confidence intervals

ABSTRACT

This paper forms the second in a short sequence considering the system identification problem for hysteretic systems. The basic model for parameter estimation is assumed to be the Bouc–Wen model as this has proved particularly versatile in the past. Previous work on the Bouc–Wen system has shown that the system response is more sensitive to some parameters than others and that the errors in the associated parameter estimates vary as a consequence. The first objective of the current paper is to demonstrate the use of a principled Bayesian approach to parameter sensitivity analysis for the Bouc–Wen system. The approach is based on Gaussian process emulation and is encoded in the software package GEM-SA. The paper considers a five-parameter Bouc–Wen model, and the sensitivity analysis is based on data generated by computer simulation of a single-degree-of-freedom system. The second major objective of the paper is also concerned with uncertainty analysis and considers the problem of obtaining estimates of parameter confidence intervals from optimisation-based system identification schemes. Two different estimators of the parameter covariance matrix are demonstrated and the results are compared with those from an independent MCMC (Markov Chain Monte Carlo) identification method.

1. Introduction

This paper represents the second in a sequence of three discussing various aspects of the system identification problem for hysteretic systems i.e. systems with memory. As discussed in the first paper [1], an extremely versatile parametric form for the modelling of hysteretic systems is provided by the Bouc–Wen model [2,3]. (As stated in [1], the literature on the Bouc–Wen model is extensive and the reader is referred to the recent book [4] for a comprehensive guide to the literature.) One of the main problems associated with adopting the Bouc–Wen form for a system model is the identification of the model parameters; the problem is complicated considerably by nonlinearity and the presence of unmeasured states [5]. In the first paper in this sequence, it was shown that it is possible to frame the identification problem in terms of an optimisation problem which is amenable to solution by the use of evolutionary algorithms.

There are a number of problems associated with the identification problem. One difficulty is associated with the fact that the response of the given hysteretic system will generally be more sensitive to some parameters than others, and these parameters will as a result, be estimated with lower accuracy. In a sense, this does not raise a problem. If the model is being developed solely as an effective representation of the system which will only be used to make predictions in

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0888-3270/$ - see front matter
doi:10.1016/j.ymssp.2012.01.005

similar operating conditions to those in which the identification data were obtained, then the insensitivity to the parameters means that any errors will have little effect. However, if the purpose of the model is truly to infer the underlying physics in some sense, then the parameter values matter. More importantly, if a model is used to extrapolate, i.e. to make predictions under conditions removed from those in which identification data were acquired, then the system may become sensitive to a different parameter set. If can also become important to estimate how uncertainty in the parameters propagates through to uncertainty in the response. Because of these issues, the problem of system identification becomes entwined with problems of uncertainty analysis (UA).

In its broadest terms, UA is concerned with quantifying output uncertainty given certain input uncertainties; this in turn requires some qualitative means of assigning a degree of uncertainty to a given situation. The most popular and longest-standing uncertainty theory is probability theory and this will provide the means for uncertainty analysis in this paper. A sub-problem of considerable interest within UA is sensitivity analysis (SA). This determines how individual input parameters are responsible for uncertainties in the output. Saltelli [6] divides SA approaches into three categories; in increasing order of power one has:

1. The lowest-level analysis is *screening*, which simply ranks the inputs in order of their importance in affecting the output. This can help a model builder identify the set of most important inputs, as well as any inputs that contribute very little to the output (and can thus be eliminated from subsequent uncertainty analyses). The main drawback of screening is that it offers no quantification of effects beyond ranking.
2. The next level, *local SA*, analyses and quantifies the effects of varying input parameters, but only around their immediate locality. This does not account for nonlinear responses however, so is of limited use in complex models. The idea of local sensitivity analysis will be familiar to structural dynamicists in the context of finite-element model-updating [7]; in that case it is possible to compute analytically the derivative of an output with respect to specified inputs. The local nature of the process is obvious from the use of calculus.
3. The most informative analysis is *global SA*, which investigates and quantifies uncertainties over the complete range of input space. Of course, this comes with the drawback of increased computational expense. In order to truly sample the complete range of possible parameters, one always has recourse to Monte Carlo analysis [8], but this can be the most expensive option of all. If a single sample is acquired on the basis of a computationally expensive run of a large model, the accumulation of many samples for Monte Carlo analysis may not be feasible, certainly not if the dimension of the input set is high.

As discussed above, SA can be desirable for a number of reasons, most often to identify the most influential inputs whose uncertainty must be reduced in order to decrease output variance; however also for design optimisation, and model simplification—i.e. identifying parameters that have little or no effect on the output and can thus be discounted. A new approach which addresses the problem of the computational expense of global SA is the use of Bayesian statistics. By creating an *emulator* (or *surrogate model* or *metamodel*) of the model under investigation, through fitting a Gaussian process to the response surface, sensitivity analysis data can be inferred at a greatly reduced computational cost with little loss of accuracy. Computational savings can be up to one or two orders of magnitude [9]. The Bayesian approach is used in this paper to carry out a sensitivity analysis for the Bouc–Wen system. In fact, the outputs of that model are not expensive to acquire; however, there are other advantages to the Bayesian approach which make it an interesting one to consider.

Briefly stated, the second major objective of the paper is also concerned with UA. As discussed a little earlier, an optimisation approach based on Differential Evolution and its self-adaptive variants has proved very effective in overcoming the problems associated with Bouc–Wen systems (nonlinearity in the parameters and the presence of unmeasured states) [1]; however, unlike system identification based on linear (and nonlinear) least-squares, this approach does not yield an estimate of the covariance matrix for the parameter estimates and therefore does not directly give error bounds. In the latter part of the current paper it is shown that meaningful error bounds for the Bouc–Wen parameter estimates can be computed based on existing estimation theory and that the results are consistent with another identification method.

The layout of the paper is as follows: Section 2 briefly describes the Bouc–Wen model of hysteresis considered here and explains the evolutionary approach to system identification. Section 3 discusses the theoretical basis of the Bayesian sensitivity analysis, including the background for the Gaussian process models which are used for the emulator. Section 4 presents the results of the sensitivity analysis for the Bouc–Wen system. Section 5 presents two estimators for parameter covariance matrices and shows how parameter confidence intervals can be estimated in optimisation-based identification schemes. The paper concludes with a little discussion in Section 6.

2. Identification of the Bouc–Wen hysteresis model

As discussed in [1] the system of interest here will be the general Single-Degree-of-Freedom (SDOF) hysteretic system described in the terms of Wen [3], it is represented below with $z(y,\dot{y})$ the hysteretic part of the restoring force,

$$m\ddot{y}+c\dot{y}+ky+z(y,\dot{y})=x(t) \tag{1}$$