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Strict sensitivity analysis in fuzzy quadratic programming

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Abstract

Quadratic programming can be seen both as a general approach to linear programming and a special class of non-linear programming. Moreover, quadratic programming problems are of utmost importance in an increasing variety of practical fields, such as, regression, efficient production and portfolio selection. As ambiguity and vagueness are natural and ever-present in real-life situations requiring solutions, it makes perfect sense to attempt to address them using fuzzy quadratic programming problems. The fxmain purpose of this paper is to study the strictly sensitivity analysis for fuzzy quadratic programming when simultaneously and independently variations occur in the right-hand-side of the constraints and the coefficients of the objective function. One presents computable auxiliary problems to identify the invariance intervals and give a fuzzy quadratic form of the optimal value function too. Some numerical examples are presented to illustrate the proposed method.

Keywords: Fuzzy quadratic programming; Trapezoidal fuzzy number; Ranking function; Fuzzy linear programming

1. Introduction

Nowadays Fuzzy Sets and Systems are applied with great success in a wide range of areas including the conception, design, and construction of practical models that simulate human reasoning. This is specifically true in the case of optimization problems, and particularly so in Mathematical Programming problems. Mathematical Programming helps to solve problems that involve minimization (or maximization) of the objective function in a function domain that can be constrained or not. Quadratic programming (QP) represents a special type of Mathematical Programming where the objective function is a quadratic function and the constraints are linear. This set of problems can be formalized in the following form:

$$\begin{aligned} \min \quad & c^t x + \tfrac{1}{2} x^t Q x \\ \text{s.t.} \quad & Ax \geq b, \\ & x \geq 0, \end{aligned}$$

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where $x \in R^n, c \in R^n, b \in R^m, A \in R^{m\times n}$ is an $m \times n$ full rank matrix, and $Q \in R^{n\times n}$ is an $n \times n$ symmetric and positive semi-definite matrix. Thus, a classical linear programming problem is a classical QP problem where $Q = 0$.

Notable applications of QP are to be found in the fields of scheduling, planning and flow computations, which can be used to solve some interesting combinatorial optimization problems. QP is also considered to be a type of non-linear programming. There are several classes of problem that are naturally expressed as QP problems. Such problems arise in various frameworks such as regression, efficient production, portfolio selection, game theory, engineering modeling, design and control. Moreover, there are some cases where the parameters of the real-world problems are not precisely known and thus have to be estimated by the decision maker. In this context, in recent years the application of Soft Computing, and Fuzzy Logic in particular, has been shown to have tremendous potential for modeling systems that are non-linear, complex, ill-defined or poorly understood.

Fuzzy Logic is a way to describe this vagueness mathematically. It has found numerous different applications due to its ease of implementation, flexibility, tolerance of imprecise data, low-cost implementation, and ability to model the non-linear behavior of arbitrary complexity thanks to its basis in natural language. Vagueness can be found in the constraints, coefficients, decision variables or all parameters of a problem. Some authors have applied Soft Computing methodologies to QP, as can be seen in [1–4,6,9,12,13,17,18]. Recently, Liu [10] has studied a solution method for solving a type of fuzzy QP, where the parameters – that is, c, b, A and Q – in the problem are fuzzy numbers. Based on Zadeh's Extension Principle [16], the fuzzy QP problem is transformed into a pair of two-level mathematical programs to calculate the upper and lower bounds of the objective value at possibility level α.

Although several approaches have been put forward for the solution of fuzzy QP, the sensitivity analysis for such problems remains to be carried out. The present paper represents a step in this direction, seeking to ascertain how the problem and the optimal solutions change under variations in the input data. Such variations occur due to calculation errors or simply when answering the "what if?" system management questions that emerged shortly after the simplex method was introduced, when the related research area was known as basic sensitivity analysis. The primary aim of sensitivity analysis is to identify the interval where the given basic optimal solution is invariant. However, since sensitivity analysis using an optimal basis cannot be applied to an optimal non-basic solution, another method has been put forward, namely strict sensitivity analysis. In this context, the aim is to find the range of parameter variations where for each parameter value in the range, an optimal solution exists with exactly the same set of positive variables as for the current optimal solution.

Luozhong et al. [8] analyzed a fuzzy QP problem defined as a QP problem with fuzzy coefficients. To solve it, they defined a crisp model equivalent to the original fuzzy QP problem, and then used the optimal solution of this model as the optimal solution of the QP problem with fuzzy coefficients. In the present work, the method used in [8] is used to investigate the sensitivity analysis of fuzzy QP and provide some fuzzy linear programming problems for computing the invariance intervals. To this end, we first transform the fuzzy linear programming problem into a new crisp linear programming problem by using a ranking function. We then put forward the key results of our work relating to conventional QP. These results validate our method to be applied to conventional QP problems, which in this case are considered a particular type of fuzzy QP problem. We also use the fuzzy simplex method in [11] for solving the fuzzy linear programming problems introduced in this paper. On this basis we study strict sensitivity analysis and obtain the fuzzy quadratic optimal value function using the ranking function in invariance intervals. Berkelaar et al. [5] obtained a quadratic optimal value function in the invariance interval of the optimal partition of the conventional QP problems. Here we present fuzzy auxiliary problems to identify the invariance interval of the optimal partition in the fuzzy case, which can also be used to calculate this interval in the specific case of conventional problems.

It is worth noting that in this work an optimal value of the objective function is obtained as a fuzzy number depending on the parameter. Thus the membership of this fuzzy number will depend on the parameter from which we can select the best value with membership closest to one, which is not possible in sensitivity analysis for a crisp case.

The present paper is organized as follows: in Section 2, an initial outline of fuzzy numbers and ranking functions is provided; Section 3 describes some basic concepts and the perturbed fuzzy QP problem; Section 4 studies the strict sensitivity analysis of the perturbed fuzzy QP problem, and examines some fundamental properties and computable methods that are used to ascertain the invariance intervals; in Section 5, we obtain a closed form of the optimal value function which is a fuzzy quadratic function with fuzzy coefficients; and finally to help clarify the latter, some numerical examples are analyzed in Section 6.