International Journal of Fatigue 40 (2012) 61–73

Contents lists available at SciVerse ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

Fatigue sensitivity analysis using complex variable methods

A. Voorhees [a,1], H. Millwater [a,*], R. Bagley [a,1], P. Golden [b,2]

[a] The University of Texas at San Antonio, San Antonio, TX 78249, United States
[b] Air Force Research Lab, RXLMN, Wright Patterson Air Force Base, Dayton, OH 45433, United States

ARTICLE INFO

Article history:
Received 14 August 2011
Received in revised form 30 December 2011
Accepted 1 January 2012
Available online 24 January 2012

Keywords:
Fatigue life prediction
Sensitivity methods
Complex Taylor series expansion

ABSTRACT

The sensitivity of the computed cycles-to-failure and other lifing estimates to the various input parameters is a valuable, yet largely unexploited, aspect of a fatigue lifing analysis. Two complex variable sensitivity methods, complex Taylor series expansion (CTSE) and Fourier differentiation (FD), are adapted and applied to fatigue analysis through the development of a complex variable fatigue analysis software (CVGROW). The software computes the cycles-to-failure and the sensitivities of the computed cycles-to-failure with respect to parameters of interest such as the initial crack size, material properties, geometry, and loading. The complex variable methods are shown to have advantages over traditional numerical differentiation in that more accurate and stable first and second order derivatives are obtained using CTSE and more accurate and stable higher order derivatives are obtained using FD.

1. Introduction

The fatigue of structural components due to repeated loading cycles is a detrimental and dangerous problem. Structural failure due to fatigue can lead to costly and time consuming repairs, early retirement or catastrophic failure of structural and mechanical systems. Therefore, the calculation of the estimated "life" of a component or a structure is a critical element in a fracture control plan. As a result, there are a number of in-house and commercial computer codes such as AFGROW [1], NASGRO [2], and DARWIN [3] for fatigue crack growth evaluation. These codes compute an estimate of the cycles-to-failure of a cracked geometry under load by integrating the crack growth rate equation until failure occurs, with failure typically defined as net-section-yield or unstable fracture. Other ancillary outputs such as crack size and the stress intensity factor as a function of loading cycles, the residual strength, and the critical crack size are also provided. The inputs that are traditionally considered are the initial crack size, the applied loading, the geometry, and the material properties, and, in some cases, inspection schedules and probability-of-detection curves.

The sensitivity of the computed cycles-to-failure to the various input parameters is a valuable, yet largely unexploited, aspect of a fatigue analysis. The fatigue analysis often contains: (a) a significant uncertainty or variability in the input parameters; (b) simplifications in the modeling such as a simplified geometry and loading; and (c) numerical approximations, e.g., when computing the geometry correction factor using handbook solutions or curve fits to numerical results such as weight functions. Sensitivity analysis can convey the significance of the various inputs on the computed cycles-to-failure. For example, the sensitivities express the expected change in the computed cycles-to-failure given a small change in the initial crack size, the part geometry, the loading cycles (gust, maneuver, ground-air-ground), the material properties (crack growth rate, fracture toughness), etc. If a sensitivity is relatively large, then a more thorough data collection effort or thorough analysis may be warranted. As a result, methods that determine the sensitivity of the cycles-to-failure to the inputs are valuable additions to a fatigue analysis.

Both deterministic and probabilistic approaches can be applied for sensitivity analysis. Deterministic studies to determine the partial derivative of the cycles-to-failure with respect to each input of interest can be obtained using numerical finite differencing.

McGinty discussed the sensitivities of damage tolerance analysis (DTA) elements by examining the deterministic ratio of the percent change of output to the percent change of input, i.e., the nondimensionalized derivative of the governing equation [4]. The following studies were developed: sensitivity of the stress intensity factor to the stress intensity geometry correction factor (β), the applied stress, and the crack size; sensitivity of the crack growth rate to β, Paris coefficient, Paris exponent, and crack size; sensitivity of the fatigue life to β, Paris coefficient, Paris exponent, and initial and final crack sizes. The results indicated that the geometry correction factor (β) and the applied stress are the important factors with respect to fatigue life.

* Corresponding author. Address: Department of Mechanical Engineering, 1 UTSA Circle, San Antonio, TX 78249, United States. Tel.: +1 210 458 4481; fax: +1 210 458 6504.

E-mail address: harry.millwater@utsa.edu (H. Millwater).

[1] Department of Mechanical Engineering, 1 UTSA Circle, San Antonio, TX 78249, United States

[2] Metals, Ceramics & NDE Division, 2230 Tenth St, Ste 1, United States

0142-1123/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijfatigue.2012.01.016

Nomenclature

a_n	Fourier series coefficients	FFT	fast Fourier transform
a_0	initial crack depth	h	step size or sampling radius of numerical differentiation
c_0	initial crack length	i	square root of negative one
C_1	Paris constant for region I	m_1	Paris exponent for region I
C_2	Paris constant for region II	m_2	Paris exponent for region II
CD	central differencing	N	number of terms in truncated Fourier series
CTSE	complex Taylor series expansion	N_f	number of cycles to failure
da/dN	crack propagation rate	Δc	stride of the differential equation solver
DTA	damage tolerance analysis	ΔK	stress intensity factor
FD	Fourier differentiation	β	stress intensity geometry correction factor

Fawaz and Harter studied the impact of various parameters on DTA estimates using deterministic parameter studies [5]. The expected variation in the parameters was estimated using engineering judgment. The cycles-to-failure was used as a metric of importance. Five different cracking scenarios were studied of a transport aircraft fuselage crack under remote tension: a single through crack at a hole, a double through crack at a hole, a single corner crack at a hole, a double corner crack at a hole, and a double oblique through crack at a hole. The material properties and their variation were chosen as representative of 2024-T3 aluminum sheet. Eight parameters were deemed important and, per the analysis results, divided into first and second order effects. The first order parameters were the initial flaw size, the geometry correction factor, the load interaction model, the crack growth rate data, and the stress intensity factor. The secondary parameters were the yield stress, fracture toughness, and threshold stress intensity factor. The distinction between first and second order effects was based on whether the life-cycle costs could be reduced via a more appropriate inspection schedule or if flight safety was affected. The parameters within each category were not comparatively ranked.

Millwater and Wieland considered the probabilistic sensitivities of a T-38 wing (corner crack growing from a fastener hole) with respect to initial crack size and aspect ratio, hole diameter, crack growth rate, hole edge distance, geometry correction factor, fracture toughness, retardation and loading spectrum [6]. The conclusion from their study was that the probability-of-failure was sensitive to the fastener hole diameter, β, the crack growth rate at higher ΔK's, and the stress spectrum scale factor. Of these variables, only the geometry correction factor was sensitive to variation in both its mean and standard deviation.

Probabilistic approaches, while more comprehensive than deterministic approaches, are more arduous since significantly more data are required to determine the probability distributions of the inputs and the analysis is more time consuming to execute. Also, the results typically identify, *after the fact*, unimportant variables for which the data collection effort was not warranted. Therefore, fast yet accurate deterministic sensitivity methods have an important role to play.

Numerical finite differencing is a straightforward commonly-used method to evaluate the derivatives of implicit functions. The method is simple in concept: change a parameter, rerun the analysis and determine the ratio of change in cycles-to-failure to the change in the input parameter. However, an estimate of the derivative is only accurate when the step size is small. On the other hand, when subtracting near-equal numbers, machine round off can also introduce error. As a result, the method is sensitive to the step size, which cannot be too large or too small. This becomes even more of a problem when calculating higher order derivatives since more subtraction operations are required. In summary, finite differencing, while easy to do, is prone to numerical issues that may be difficult to discern. It is typical that a laborious trial-and-error effort is required to locate a step size that results in a satisfactory derivative.

Alternatives to finite differencing methods are complex variable sensitivity methods, in particular, complex Taylor series expansion (CTSE) and Fourier differentiation (FD). CTSE was originally described by Lyness and Moler [7,8] and was brought to the attention of the engineering community by Squire and Trapp [9]. In CTSE, the first derivative is calculated by perturbing the parameter of interest along the imaginary axes. For example, the initial crack size is given an imaginary perturbation, e.g., $a_0 + ih$, where i denotes an imaginary number and h the step size. As derived below, the sensitivity is then estimated by evaluating the imaginary component of the cycles-to-failure and dividing by the step size. Consequently, CTSE involves no difference operations, thus, allowing for the step size to be made arbitrarily small. Hence the issue of choosing an accurate step size is avoided, making CTSE an easy to implement and highly accurate method for the numerical calculation of first derivatives. In order to calculate the higher order derivatives using CTSE, additional sample points along the imaginary axis are needed, then, in this case, differencing operations are necessary and the step size must be chosen carefully.

Fourier differentiation (FD) is analogous complex variable sensitivity method for determining higher order derivatives using sampling in the complex plane. This technique was first described by Lyness et al. [7,8] FD requires the evaluation of sample points along a circular contour around the initial point in the complex plane and a fast Fourier transform (FFT) of the evaluated sample points is used to calculate the derivatives. The use of the FFT as a method to calculate derivatives was described by Lyness et al. [10] and by Henrici [11] and more recently by Bagley [12] to compute of sensitivities of implicit functions. The advantage of FD is that it can compute higher order derivatives more accurately than CTSE or finite differencing. The number of derivatives that can be obtained from the FFT is related to the number of sample points chosen.

CTSE has been applied in several engineering fields but as yet is not widely known nor applied. In fluid dynamics, CTSE has been used to find sensitivities for the solution of the Navier–Stokes equation [13]. Furthermore, researchers have been able to apply CTSE techniques to finite element methods in the field of aerodynamics and aero-structural analysis [14,15]. CTSE has also been applied in the study of heat transfer [16], dynamic system optimization [17], pseudospectral [18] and eigenvalue sensitivity methods [19]. CTSE has been compared with automatic differentiation and shown that it is equivalent to the forward mode with comparable accuracy and much simpler implementation [20]. CTSE has been implemented within a solid mechanics finite element program to compute shape sensitivities with good accuracy and it was shown that the accuracy of the derivatives were a function