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# Mehar's method for solving fuzzy sensitivity analysis problems with *LR* flat fuzzy numbers

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ABSTRACT

In published works on fuzzy linear programming there are only few papers dealing with stability or sensitivity analysis in fuzzy mathematical programming. To the best of our knowledge, till now there is no method in the literature to deal with the sensitivity analysis of such fuzzy linear programming problems in which all the parameters are represented by *LR* flat fuzzy numbers. In this paper, a new method, named as Mehar's method, is proposed for the same. To show the advantages of proposed method over existing methods, some fuzzy sensitivity analysis problems which may or may not be solved by the existing methods are solved by using the proposed method.



## 1. Introduction

The fuzzy set theory is being applied massively in many fields these days. One of these is linear programming problems. Sensitivity analysis is well-explored area in classical linear programming. Sensitivity analysis is a basic tool for studying perturbations in optimization problems. There is considerable research on sensitivity analysis for some operations research and management science models such as linear programming and investment analysis.

In most practical applications of mathematical programming the possible values of the parameters required in the modeling of the problem are provided either by a decision maker subjectively or a statistical inference from the past data due to which there exists some uncertainty. In order to reflect this uncertainty, the model of the problem is often constructed with fuzzy data [1].

Fuzzy linear programming provides the flexibility in values. But even after formulating the problem as fuzzy linear programming problem, one cannot stick to all the values for a long time or it is quite possible that the wrong values got entered. With time the factors like cost, required time or availability of product etc. changes widely. Sensitivity analysis for fuzzy linear programming problems needs to be applied in that case. Sensitivity analysis is one of the interesting researches in fuzzy linear programming problems.

Zimmermann [2] attempted to fuzzify a linear program for the first time, fuzzy numbers being the source of flexibility. Zimmermann also presented a fuzzy approach to multi-objective linear programming problems and its sensitivity analysis. Sensitivity analysis in fuzzy linear programming problem with crisp parameters and soft constraints was first considered by Hamacher et al. [3].

Tanaka and Asai [4] proposed a method for allocating the given investigation cost to each fuzzy coefficients by using sensitivity analysis. Tanaka et al. [5] formulated a fuzzy linear programming problem with fuzzy coefficients and the value of

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information was discussed via sensitivity analysis. Sakawa and Yano [6] presented a fuzzy approach for solving multi-objective linear fractional programming problems via sensitivity analysis.

Fuller [7] proposed that the solution to fuzzy linear programming problems with symmetrical triangular fuzzy numbers is stable with respect to small changes of centers of fuzzy numbers. Perturbations occur due to calculation errors or just to answer managerial questions "What if ...". Such questions propose after the simplex method and the related research area refers to as basis invariance sensitivity analysis.

Dutta et al. [8] studied sensitivity analysis for fuzzy linear fractional programming problem. Verdegay and Aguado [9] proposed that in the case of fuzzy linear programming problems, whether or not a fuzzy optimal solution has been found by using linear membership functions modeling the constraints, possible further changes of those membership functions do not affect the former optimal solution. The sensitivity analysis performed for those membership functions and the corresponding solutions shows the convenience of using linear functions instead of other more complicated ones.

Gupta and Bhatia [10] studied the measurement of sensitivity for changes of violations in the aspiration level for the fuzzy multi-objective linear fractional programming problem. Precup and Preitl [11] performed the sensitivity analysis for some fuzzy control systems. Lotfi et al. [12] developed a sensitivity analysis approach for the additive model. Kumar, et al. [13] pointed out the shortcomings of the existing method [14] and proposed a method to find the fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints.

Kheirfam and Hasani [15] studied the basis invariance sensitivity analysis for fuzzy linear programming problems. Ebrahimnejad [16] generalized the concept of sensitivity analysis in fuzzy number linear programming problems by applying fuzzy simplex algorithms and using the general linear ranking function on fuzzy numbers. Nasseri and Ebrahimnejad [17] proposed a method for sensitivity analysis on linear programming problem with trapezoidal fuzzy variables.

In this paper, the limitations of existing works [16,15,17] are pointed out. To overcome these limitations a new method, named as Mehar's method, is proposed to deal with the sensitivity analysis of such fuzzy linear programming problems in which all the parameters are represented by *LR* flat fuzzy numbers. To show the advantages of proposed method over existing methods, some fuzzy sensitivity analysis problems which may or may not be solved by the existing methods are solved by using the proposed method.

This paper is organized as follows: In Section 2, some basic definitions, arithmetic operations and Yager's ranking approach for comparing *LR* flat fuzzy numbers are presented. In Section 3, the limitations of existing methods are pointed out. In Section 4, a new method, named as Mehar's method, is proposed to deal with the sensitivity analysis of such fuzzy linear programming problems in which all the parameters are represented by *LR* flat fuzzy numbers. Advantages of proposed method over the existing methods are discussed in Section 5. Results are discussed in Section 6. Finally we conclude in Section 7.

## 2. Preliminaries

In this section, some basic definitions, arithmetic operations of *LR* flat fuzzy numbers and an existing ranking approach for comparing *LR* flat fuzzy numbers are presented.

### 2.1. Basic definitions

In this section, some basic definitions are presented [18].

**Definition 2.1.** A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be an *LR* flat fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leqslant m, \alpha > 0, \\ R\left(\frac{x-n}{\beta}\right), & x \geqslant n, \beta > 0, \\ 1, & m \leqslant x \leqslant n. \end{cases}$$

If $m = n$ then $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ will convert into $\tilde{A} = (m, \alpha, \beta)_{LR}$ and is said to be an *LR* fuzzy number. *L* and *R* are called reference functions, which are continuous, non-increasing functions that defining the left and right shapes of $\mu_{\tilde{A}}(x)$ respectively and $L(0) = R(0) = 1$. Two special cases are triangular and trapezoidal fuzzy number, for which $L(x) = R(x) =$ maximum $\{0, 1 - x\}$, are linear functions. Three commonly used nonlinear reference functions with parameters $q$, denoted as $RF_q$, are summarized as follows:

power: $RF_q =$ maximum $\{0, 1 - x^q\}$, $q \geqslant 0$,
exponential power: $RF_q = e^{-x^q}$, $q \geqslant 0$,
rational: $RF_q = \frac{1}{(1+x^q)}$, $q \geqslant 0$.

**Definition 2.2.** A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be a non-negative *LR* flat fuzzy number if $m - \alpha \geqslant 0$ and is said to be a non-positive *LR* flat fuzzy number if $n + \beta \leqslant 0$.