Comput. Methods Appl. Mech. Engrg. 245–246 (2012) 75–89

Contents lists available at SciVerse ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Integrated layout design of multi-component systems using XFEM and analytical sensitivity analysis

J. Zhang, W.H. Zhang *, J.H. Zhu, L. Xia

Engineering Simulation and Aerospace Computing (ESAC), School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China

ARTICLE INFO

Article history:
Received 27 March 2012
Received in revised form 27 June 2012
Accepted 28 June 2012
Available online 7 July 2012

Keywords:
Shape sensitivity analysis
Layout optimization
Topology optimization
The extended finite element method
Multi-component system

ABSTRACT

This study presents the integrated layout optimization of multi-component systems using a fixed mesh. The optimization formulation is established under the framework of the extended finite element method (XFEM). The level set method is used to represent components and is combined with the XFEM to describe material discontinuities across elements. Sensitivity analysis is proposed with respect to geometric variables of components and pseudo-densities of the basic structure. An analytical shape sensitivity analysis method with respect to positions and shapes of components is developed. Both solid and void components are considered to show the efficiency and accuracy of the proposed shape sensitivity analysis method. Furthermore, a revised finite circle method that adapts shape changes of elliptical components is proposed for the definition of non-overlapping constraints. Finally, numerical examples of maximizing the structural stiffness are tested to demonstrate the proposed method.

1. Introduction

Structural optimization, including topology, shape and size optimization, has been widely used in industrial and civil engineering to improve the performance of mechanical designs. As illustrated in Fig. 1, the integrated layout optimization studied here aims to find a proper position and shape design of components and the topology of the base structure in a limited base domain simultaneously. Embedded components may be considered as solids or substructures to afford loads or as voids to reduce the overall structure weight. Three kinds of interfaces thus exist between the base structure and the components, i.e., material–material (m–m) interface, material–void (m–v) interface and void–void (v–v) interface.

In this paper, as the void part of the base structure is considered to be a special domain filled with materials of very weak stiffness as in traditional topology optimization [1], attention is paid only to the first kind of interface (m–m) that can be regarded as a generalization of the rest two ones. Historically, components were designed as a packing optimization problem in the previous work [2–6] where only their positions were changeable without shape variation. The current work is an extended approach that combines the packing optimization and shape optimization together for more design flexibilities of integrated layout optimization.

Clearly, there exist two main concerns in the integrated layout design of multi-component systems as discussed by Zhang et al. [6]. First, the avoidances of overlap between components and overlap between the component and the domain boundary. Second, shape sensitivity analysis with respect to geometric design variables related to positions and shapes of components.

The overlapping avoidance is a very important issue for optimization of multi-component systems. In the earlier work of Qian and Ananthasuresh [2], each component was approximated by one circle so that the overlapping judgment between components turned out to determine the distance between two circles. However, when the component has a complex shape, especially concave, this approximation is very rough. To improve the approximation accuracy, Zhang and Zhang [7] proposed the so-called finite circle method (FCM) by which each component is approximated with a set of circles and non-overlapping constraints between components are thus converted into simple constraints between two sets of circles.

Another underlying difficulty concerns shape sensitivity analysis related to the shape representation of the component. Explicit parametric representations, e.g., Bézier or NURBS [8], implicit representations, e.g., the signed distance function [9] or R-function [10] and combined representations [11] can be used. To the authors' knowledge, there exist mainly three sensitivity analysis methods related to different shape representations and different mesh discretizations. Fig. 2 illustrates the case of a movable void component inside a meshed plate.

(a) Grid perturbation method using Lagrangian mesh. As shown in Fig. 2a, the FE mesh will move in accordance with the variation of the component boundary that is often parameterized to define geometric design variables. This implies that the model is based

* Corresponding author. Tel./fax: +86 (0) 29 88495774.
E-mail address: zhangwh@nwpu.edu.cn (W.H. Zhang).

0045-7825/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.cma.2012.06.022

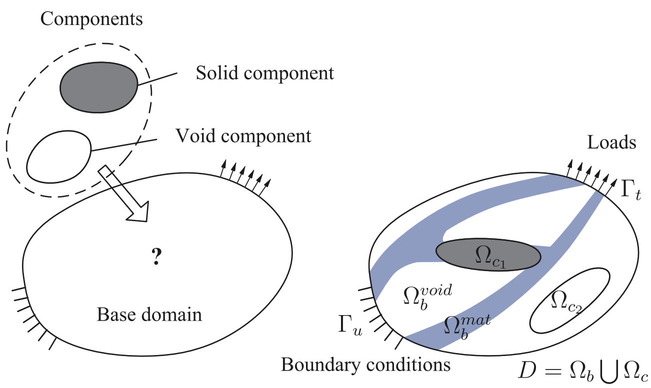

Fig. 1. Illustration of the integrated layout optimization.

on a Lagrangian mesh that varies when geometric design variables are perturbed or modified in shape sensitivity analysis and optimization process, respectively. As a result, the computational domain has to be remeshed for any change of geometric design variables [12]. This kind of method always works in conjunction with a mesh morphing technique [13] to relocate the nodes in sensitivity analysis while the mesh topology is kept unchanged.

(b) Material projection method using Eulerian mesh. In order to avoid the sophisticated remeshing procedure, a fixed Eulerian mesh is used for the base domain independently of the shape variation of the component boundary. In this case, an implicit shape representation approach, e.g., level set method [9] is often used for the geometric description of the component boundary. To characterize the effect of boundary variation, the fixed mesh will, however, undergo a change of attributed material properties depending upon the location of each element. As illustrated in Fig. 2b, material attributes stay unchanged for inner elements that do not share the component boundary, while intermediate values of material attributes have to be assigned to boundary elements.

A simple way is to take the area fraction of the solid part in the boundary elements for the correction of material properties, as discussed in the work of Allaire et al. [14], Kim and Chang [15] and Luo et al. [16,17]. However, the stress sensitivity is still questionable. Another way to make one such correction is the so-called filtering technique as used in the work of Norato et al. [18]. In the work of Belytschko et al. [19] and Wang et al. [20], an approximate Heaviside function was used for the correction. However, this kind of method might cause low accuracy for analysis results near the component boundary because the analysis model is not the same as the exact boundary shape representation [21]. Meanwhile, a rather fine mesh is sometimes needed.

(c) The XFEM based method using Eulerian mesh. This method is based on a fixed mesh that works with the level set method firstly proposed by Osher and Sethian [9] to represent moving interfaces and has been extensively applied to structural optimization [14,20,16,17,22]. The level set-based structural optimization method is essentially a type of shape optimization method but able to achieve topological changes during shape variations, which can flexibly describe complex geometries via topological shape evolvements by remaining a concise and smooth boundary. Until now, several level set methods have been proposed for the layout description of multi-phase materials. In the standard level set method [9], components of the same material are represented using one level set function. In the work of Wang and Wang [23], a color level set method which represents n different material phases uses $log_2 n$ level set functions by allowing overlap of the level set functions. Luo et al. [24] proposed a piecewise level set method using one level set function of constant integer value for each material phase. The representation of multi-phase materials is fairly easy by making use of discontinuities of the piecewise level set function. Notice that the piecewise level set function is non-differentiable at material interfaces. As the overlap of components is avoided by FCM and the differentiability of the level set function is needed in sensitivity analysis, the standard level set method is employed in this work and combined with the XFEM [25–27] for computing structural responses and sensitivities. As shown in Fig. 2c, whether an embedded void or solid component is concerned, the whole base domain is completely meshed. Material discontinuities within each element are handled by the XFEM based on the partition of unity method (PUM) [28]. As a result, the shape representation of the component boundary and the analysis model are unified to increase the accuracy of analysis results.

As the finite element approximation is locally enriched by the XFEM, special treatments in shape sensitivity analysis should be considered correspondingly. Miegroet and Duysinx [29] adopted the semi-analytical method (SAM). The dimension change of the stiffness matrix of an enriched element is ignored when the design boundary is very close to a node. To some extent, this treatment might degrade the accuracy of sensitivity results. In the work of Wei et al. [21] and Shu et al. [30], shape sensitivity is calculated by using the shape derivative method. When the element stiffness matrix is calculated by the XFEM, a simplified strategy is used by removing the integration points on the void part of an enrichment element without additional degrees of freedom. As indicated by Daux et al. [31] and Miegroet and Duysinx [29], the inconvenience lies in that this simplification is only suitable to material–void

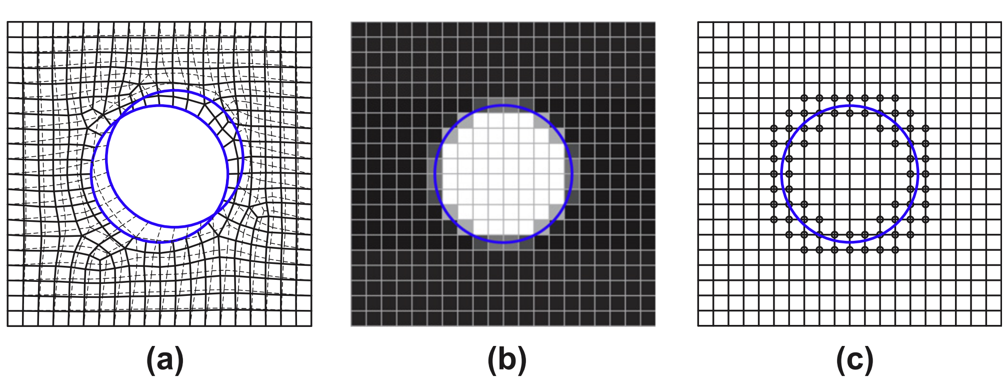

Fig. 2. Three methods for shape sensitivity analysis of a void component. (a) Grid perturbation method with Lagrangian mesh. (b) Material projection method with Eulerian mesh. (c) XFEM based method with Eulerian mesh.