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Random and quasi-random designs in variance-based sensitivity analysis for partially ordered sets

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ABSTRACT

A special application of global sensitivity analysis is that on partially ordered sets – posets – that are sets of objects endowed by a binary order relation. In a partial order two objects can be in relation with each other, and are said to be comparable, or not, then they are said to be incomparable. Comparability and incomparability relations between objects can be visualized in a so called Hasse diagram, which is instructive in ranking the objects by multi-criteria/non-compensatory approaches. The interpretation of a Hasse diagram may be difficult even when the number of objects in the set is relatively small. Completely different configurations of the diagram can arise even for small perturbations of the starting data. Global sensitivity indices can shed light on the robustness of the partial order to data value uncertainty. Global measures particularly fit the case since posets are characterized by high number of dimensions and high-order interactions. These distinctive features of sensitivity analysis for posets make quasi-random designs perform almost the same as the random one as it is discussed here with a real test case for comparing the level of competitiveness of EU countries.

1. Introduction

Sensitivity analysis studies how different sources of uncertainties play into a mathematical model, and in particular how the uncertainty in a given model prediction can be parceled out to different input factors or assumptions. Variance-based indices are global methods for sensitivity analysis in contrast to local sensitivity methods which are based on first or higher-order derivatives of the output with respect to the input factors. If the model is linear, first-order derivatives provide all the information that is needed for sensitivity analysis (SA). If the model is non-linear but additive, i.e. there are no interactions among factors, then derivatives of higher and cross order will be sufficient to understand the model. However when this kind of a priori information on the nature of the model is not available (model-free setting) or the model is acknowledged to be non-additive, then global methods are needed [14]. The recommended approach in the estimation of variance-based sensitivity analysis of deterministic models – VB-SA – is to use quasi-random sequences [16,13]. These are deterministically uniformly distributed sequences that are designed to place sample points as uniformly as possible in the multi-dimensional space of the uncertain input factors. For multidimensional integration quasi-random sequences often provide rates of convergence higher than random sequences.[1] In particular while the convergence rate for random number sequences is of the order of $1/\sqrt{N}$, where N is the number of points thrown in the hyperspace to be explored, the convergence rate for quasi-random numbers can be at times as high as $1/N$. However this property of quasi-random sequences depends on the effective dimensionality of the problem [11] and is the special focus in this paper.

In this paper, we compare the performance of quasi-random and random approaches in the estimation of first and total variance-based sensitivity indices for the case of partially ordered sets (posets hereafter). Posets can be viewed as a model that receives a matrix of values as input and gives a representation of order relations between objects as output. The resulting set of order relations can in no way be formulated as equations as it is done in standard calculus $y=f(x_1,x_2,\ldots,x_k)$. Therefore a poset ia a special kind of model where no a priori information is available and must be considered as a black box. In these cases where no hints about the model structure can be inferred global methods are recommended.

As recently discussed in Annoni et al. [1], the distinctive characteristics of the problem are high number of dimensions and high-order interaction effects. The synergy of these two

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[1] The concept of random numbers is a mathematical abstraction. We use the term 'random' as a synonymous of 'pseudorandom'.

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circumstances is a model with a high effective dimensionality which overrides the benefit of quasi-random sequences with respect to random ones. A real case study is set up to verify this.

2. Material and methods

2.1. Case study: a composite indicator for EU competitiveness

To exemplify the proposed method a recent composite indicator is used which measures the level of competitiveness in the European Union. The index is the aggregated measure of different aspects – pillars hereafter – which describe the concept of competitiveness. The detailed description of the index and its weighting scheme is beyond the scope of the paper and it is provided in an open-source report by the European Commission [2]. In the following a short description of the index structure is provided.

The index builds on, *mutatis mutandis*, the methodology developed by the WEF for the Global Competitiveness Index [15] which covers a comprehensive set of social and economic aspects of territorial competitiveness. Eleven pillars describing different aspects of competitiveness are included in the index. They are classified into three major groups: (*i*) Basic, (*ii*) Efficiency and (*iii*) Innovation. The basic group includes the five pillars (1) Institutions, (2) Macroeconomic Stability, (3) Infrastructure, (4) Health and (5) Quality of Primary and Secondary Education. These five pillars represent the key basic drivers of all types of economies. As the regional economy develops, other factors related to higher skilled labor force and a more efficient labor market enter into play for its advancement in competitiveness and are part of the Efficiency group. This includes three pillars (1) Higher Education, Training and Lifelong Learning, (2) Labor Market Efficiency and (3) Market Size. At the most advanced stage of development of a regional economy, drivers of improvement are part of the Innovation group which consists of three pillars: (1) Technological Readiness, (2) Business Sophistication and (3) Innovation.

The weighting scheme of the index is designed on a stepwise procedure: multivariate analysis for consistency check within each pillar; equal weighting across pillars between each group; differential weighting scheme across the groups to take into account the different capability of each region (for further details refer to Annoni and Kozovska [2]). Each region can then be associated to three sub-indices describing Basic, Efficiency and Innovation factors of competitiveness. As the country level of competitiveness is considered here, the starting data matrix used as case study (Table 1) consists of 27 rows (EU member states) and three columns with the three sub-indices related to the pillar groups Basic, Efficiency and Innovation.

The 2010 competitiveness index is a composite indicator. Composite indicators are popular instruments to measure complex, multidimensional, latent phenomena in many different fields. They are more and more used and requested by stakeholders and policy makers to support or controvert policy decisions and possible actions. Composite indicators are obtained by aggregating a (high) number of observable variables. A debate has been going on for a long time in the scientific community about the pitfalls of summarizing a multi-dimensional concept into a single index. A good recent authority is represented by the guidelines for the measurement of economic performance and social progress [18], commissioned by the French President Nicholas Sarkozy to give indications for assessing quality of life. Stiglitz and co-authors claim that, while there is a strong demand to develop aggregate measures, quality of life can be assessed only by analyzing a plurality of aspects. Their view on composite indicators is that they should be 'better regarded as invitations to look more closely at the various components that underlie them' [18, p. 65] with major concern on the aggregation procedure. With this claim they are implicitly supporting non-compensatory methods.

Table 1
Case study: EU country competitiveness on the three sub-indices for Basic, Efficiency and Innovation pillar groups [2]. Standard deviation in the last row.

country	Indicator 1 (basic factors)	Indicator 2 (intermediate factors)	Indicator 3 (innovative factors)
BE	0.393	0.315	0.590
BG	−1.289	−0.864	−1.240
CZ	−0.146	−0.218	−0.590
DK	0.941	0.635	0.767
DE	0.494	0.223	0.591
EE	−0.026	−0.283	−0.258
IE	0.435	0.366	0.323
GR	−0.806	−0.668	−0.910
ES	−0.038	−0.276	−0.325
FR	0.101	0.097	0.335
IT	−0.285	−0.258	−0.320
CY	−0.433	−0.08	−0.643
LV	−1.032	−0.443	−0.657
LT	−0.994	−0.133	−0.735
LU	0.542	0.417	0.943
HU	−0.87	−0.472	−0.710
MT	−0.583	−1.013	−0.467
NL	1.049	0.855	0.888
AT	0.493	0.285	0.226
PL	−0.507	−0.407	−0.682
PT	−0.277	−0.493	−0.760
RO	−1.651	−0.783	−1.235
SI	0.044	0.251	−0.221
SK	−0.448	−0.559	−0.555
FI	1.498	0.361	0.747
SE	0.927	0.338	0.658
UK	0.282	0.577	0.454
Standard deviation	0.763	0.496	0.677

Partial order tools represent a valid alternative with respect to a combined index: they are fully non-compensatory, scale independent and are based solely on the order relations between objects (countries in our case).

2.2. Partial ordered sets: introductory remarks

The interest in posets and in their graphical representation for example by Hasse diagrams arises from the need of an ordinal analysis of data matrices when an evaluation of multi-indicator systems is wanted. Ordinal analysis often, even if not exclusively, aims at a ranking of objects. Ranking of objects is relatively easy if a single, quantitative score is available which describes objects with respect to the phenomenon to be measured. For example different management measures for protecting the climate may be ranked due to their estimated costs. However other factors, other then purely economic ones, often play an important role and cannot always be mapped onto a monetary scale. In a generalized approach we come up with a multi-indicator system which includes several indicators relevant for the concept to be described. There are many methods available to derive from a multi-indicator systems a final ranking. They differ in the amount of additional information needed and how far an averaging/aggregation over indicator values is accepted or not. All these methods belong to the increasing field of decision support systems.

Many decision support systems are based on comparisons among the objects. These comparisons must obey certain logical requirements as for example transitivity: if object a is better than object b, and object b is better than object c, then it is reasonable