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A sensitivity analysis for an evolution model of the Antarctic ice sheet

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ABSTRACT

The evolution of the Antarctic ice sheet for the last 200,000 years is simulated with a finite difference thermomechanical model based on the shallow ice approximation. The model depends on the surface temperature, the ice accumulation rate, the geothermal heat flux and the basal sliding coefficient, which are estimated with large uncertainty. A second-order approximation of the model in a neighborhood of the reference values for these parameters permits the computation of both local and variance-based sensitivity indices. The results show the dominant effect of the surface temperature on the model predictions.

1. Introduction

The presence of water at the base of the Antarctic ice sheet has been revealed from several geophysical surveys [1] and requires that the temperature at the bedrock is close to the melting point. The melt rate is one of the terms of the mass balance of the Antarctic ice sheet, which is a huge reservoir of (frozen) fresh water, and therefore is an important parameter to assess the impacts of climate change on the Earth. Since environmental conditions prevent from acquiring direct information, the knowledge about the physical processes occurring at the base of the Antarctic ice sheet is still incomplete. Simulation models can be used to test different hypotheses and to plan field or remote sensing surveys. In this paper it is illustrated an example of a dynamical model of the ice sheets, which is applied to simulate the evolution of the Antarctic ice sheet during the last 200,000 years before present.

Some of the input parameters might be affected by strong uncertainty, which reflects into the model outcomes. Moreover, the non-linearity of the physical processes makes it difficult to identify which parameters are the most important to obtain physically consistent results. Therefore the sensitivity analysis aims not only to quantify the reliability of the model predictions, but also to identify which parameters require a better estimate.

A thorough review of the concepts of sensitivity analysis can be found in textbooks on this topic: see, e.g., [2] for a recently updated work. Here a very short discussion of few basic ideas related to the specific test presented in this paper is given.

Local measures of uncertainty, which are essentially based on linear approximations of the model, can be easily computed even for very complex non-linear models. A more advanced approach to sensitivity analysis would require the computation of variance-based sensitivity indicators, which are more relevant for the above mentioned goals. However, such indices are difficult to be computed for complex non-linear numerical models which require not only great computing power, but also great care to avoid numerical instabilities when the code runs with inconsistent values of the input parameters.

As a first step toward a thorough sensitivity analysis, in this paper the first-order sensitivity index is analytically computed with an approximated model. In particular, the model output is approximated in a neighborhood of some input parameters as a second-order function of the deviation from their reference values.

In the next section the model characteristics are summarised and the results of its application are discussed. The third section is devoted to the application of the sensitivity analysis for this model. In the fourth section the results are illustrated and conclusive remarks are given.

2. The model

A thermomechanical ice sheet model is based on the fundamental principles of the conservation of mass, linear and angular momentum and energy, together with the appropriate constitutive

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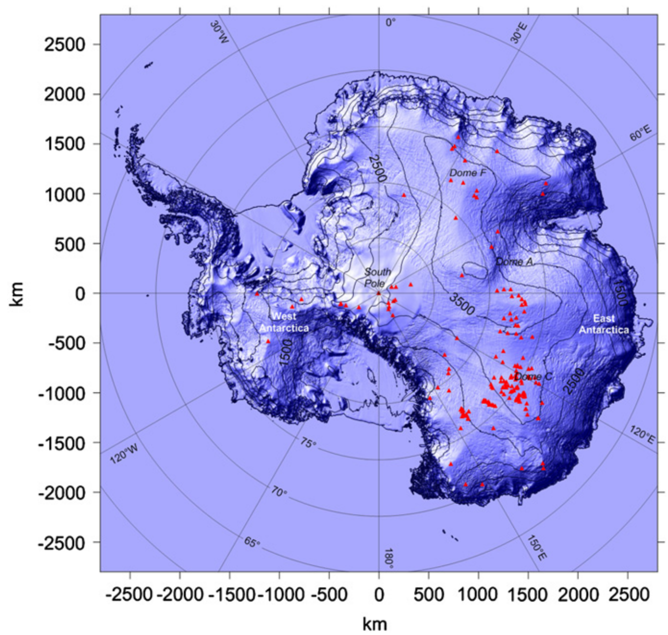

Fig. 1. Digital elevation map of Antarctica [17] (contour lines of the surface height in meters above mean sea level: equidistance 500 m) and location of subglacial lakes [1,18].

equations, which describe the thermal and rheological behaviour of ice, and with boundary conditions, which set the climatic forcings.

This model is applied to simulate the evolution of the Antarctic ice sheet (Fig. 1) during the last 200,000 years.

2.1. Basic equations

The basic equations of an ice sheet model are briefly recalled here. For a detailed discussion see, e.g., [3–5].

Assuming that the ice is incompressible, and considering proper kinematic boundary conditions at the surface and at the base (see, e.g., [6]), the mass conservation is expressed by:

$$\partial_t H = -\nabla' \cdot (H\overline{\mathbf{u}'}) + M_s - M_b, \tag{1}$$

where $\mathbf{u}' = (u,v)$ is the ice velocity in the horizontal plane, $H = s - b$ is the ice thickness (s and b are respectively the height above mean sea level of the top and bottom surface of the ice sheet), M_s is the surface accumulation rate, M_b is the basal melt rate, $\overline{\mathbf{u}'} = H^{-1}\int_b^s \mathbf{u}'(z)\,dz$ denotes the horizontal velocity averaged vertically over the ice thickness, and $\nabla' = (\partial_x, \partial_y)$.

Under the quasi-static approximation, the conservation of linear momentum reduces to the following Stokes' equations:

$$\nabla \cdot \sigma = \rho \mathbf{g}, \tag{2}$$

where σ is the stress tensor, ρ is the ice density and $\mathbf{g} = (0,0,-g)$ is the gravity acceleration.

If melting is assumed to occur only at the ice sheet base, then the temperature field is controlled by the following energy conservation equation:

$$\partial_t T = \chi \nabla^2 T - \mathbf{u} \cdot \nabla T + \Sigma, \tag{3}$$

where T is the ice temperature, χ is the thermal diffusivity of ice and Σ is the strain heating. Eq. (3) shows that the temporal variation of temperature is controlled by heat diffusion (first term of the right-hand side), heat convection (second term) and strain heating, that is the heat generated by friction during the ice deformation (third term).

The constitutive equation applied for glacier ice is usually the Glen's law (see, e.g., [4]), which states that the relation between the deviatoric stress and the strain rate is non-linear and depends on the ice temperature. On the other hand, the heat equation (3) includes convection and strain heating, so that the temperature depends on the velocity field. Therefore the five scalar equation (1)–(3) are non-linearly coupled with each other. This problem is referred to as thermomechanical coupling. In order to reduce the complexity of this non-linear thermomechanically coupled model, some common assumptions (hydrostatic conditions and SIA – Shallow Ice Approximation) have been introduced (see, e.g., [7]).

The equations have been discretised with a finite difference scheme using a Crank–Nicolson method for the time evolution; the strain heating and the convective terms are evaluated on a staggered grid with an up-wind scheme (see [8] for details). The model has been implemented with an original computer code written in FORTRAN90 and has been validated by its application to several synthetic case studies taken from the EISMINT experiments [9,10].

2.2. Boundary conditions

A stress-free boundary condition is assumed at the ice sheet surface, whereas the basal velocity is prescribed by a Weertman-type sliding law, assuming it is non-zero only when the basal temperature reaches the melting point:

$$\mathbf{u}'_\mathbf{b} = -B_s \rho g H \nabla' s, \tag{4}$$

i.e., the basal velocity is proportional, through a basal sliding coefficient B_s, to the vertical shear stresses at the base of the ice sheet.

The boundary conditions of the energy equation (3) require that the surface temperature $T_s(x,y,t)$ is assigned, while at the base it is necessary to distinguish between the situation of frozen and wet ice-bedrock interface. When the basal temperature is below the melting point ($T_b < T_{pmp}$) the basal interface is frozen and the basal velocity vanishes, so that all the geothermal heat flux G is conducted through the ice, according to the following Neumann boundary condition:

$$-\kappa \partial_z T|_{z=b} = G, \tag{5}$$

where κ is the thermal conductivity.

When the basal temperature reaches the melting point ($T_b = T_{pmp}$), instead, the heat balance at the ice-bedrock interface includes also the frictional heating and the latent heat spent to melt the ice, according to the following relation:

$$-\kappa \partial_z T|_{z=b} = G + \rho g H \mathbf{u}'_\mathbf{b} \cdot \nabla' s - \rho L_f M_b, \tag{6}$$

where L_f is the latent heat of fusion.

2.3. Input data of the model

The input data of the model include physical parameters of ice (ρ, κ, L_f, χ) which are assumed to be known and constant and some data which are estimated with greater uncertainty: the basal topography b, the surface temperature T_s, the surface accumulation rate M_s, the geothermal heat flux G and the basal sliding coefficient B_s. For these parameters we define a reference situation based on values proposed in the literature [10–14].

The basal topography is taken from [11], the surface temperature is given by [11]:

$$T_s(\mathbf{x}',t) = T_{s0}(\mathbf{x}',t) + \delta T(t) + \beta_T[s(\mathbf{x}',t) - s_0(\mathbf{x}')],$$

where t is time before present, $T_{s0}(\mathbf{x}',t)$ is an estimate of the present day surface temperature [11], $\delta T(t)$ is the time variation of the temperature for an ice core in the Vostok area (Fig. 2, from [12]), s_0 is an estimate of the present day surface of the ice sheet and β_T is the vertical gradient of the surface temperature.