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Efficient classification based methods for global sensitivity analysis

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ABSTRACT

New classification based methods for global sensitivity analysis of structural models are presented which do not require the full approximation of the model response for qualitatively good sensitivity measures. Instead, only the level sets of the model response are identified by partitioning it into a number of classes with a few available sample points. The average change in class memberships of simulated points on the model domain is considered as sensitivity measure. The new methods are realized using Support Vector Machines and their results are compared with existing methods by using analytical as well as practical industry examples.

1. Introduction

Global sensitivity analysis is a procedure to analyze the full range of plausible values of (random) parameters and their interactions in a structural model in order to assess their impact on the model response. Global in a sense that the analysis is not performed locally or one-factor-at-a-time but considering the whole domain of each model parameter. Each individual model parameter $X_{i,\ i=1,2,\dots,n}$ is compared with the other remaining parameters $X_1,\dots X_{i-1}$, $X_{i+1},\dots,X_n$ in order to evaluate its influence on the model response Y. The objective of sensitivity analysis is to identify the most significant model parameters affecting a specific model response [18]. Significant parameters are those which have a large impact on the model response. Results of the sensitivity analysis are the sensitivity measures $S_{i,i=1,2,\dots,n}$, i.e., significance of a parameter X_i is identified with reference to its sensitivity measure S_i. The normalized sensitivity measure S_i for a model parameter X_i is given as

$$S_i = \frac{\widetilde{S}_i}{\sum_{j=1}^{n} \widetilde{S}_j}, \qquad (1)$$

where $\widetilde{S}_i$ represents the influence of X_i on Y according to a specific sensitivity measure. Sensitivity measures are used for the identification of the significant parameters before the optimization of a design structure [10,14,16,17]. Optimization of design structures is a computationally extensive process. During the optimization process, the objective function which is formulated on the basis of the model response is analyzed depending on the design parameters and constraints. An optimization model is often dependent in part on the number of design parameters. The complexity of the optimization problem can be reduced if the relationship between the design parameters and the model response is effectively identified and only the significant design parameters are then used. This relationship is captured by the methods of global sensitivity analysis.

Different sensitivity measures have been proposed which are determined using variance based methods [18,19], sampling based methods [9], and derivative based methods [19]. In variance based methods, the unconditional variance of the model response is decomposed into terms due to individual factors and the terms due to the interaction among factors. Common variance based global sensitivity measures are for example ANOVA measures [18] and Sobol Indices [19]. Sobol Indices are popular because they capture the non-linearity in the models as well [16]. Sampling based methods include screening methods and correlation analysis, which is only applicable for linear problems. Derivative based methods use partial derivatives to determine the importance of the input parameters because they represent the instant slope of the underlying function for each value of the input parameter [19].

If the model is unknown, meta-model based global sensitivity methods can be used for sensitivity analysis. An approximate model is constructed for the available response values using meta-models and certain intrinsic properties of the meta-models are then used to derive sensitivity measures [22]. Based on neural networks as meta-models, different equations have been proposed which calculate the products of the weights of the network and then obtain the sum of the calculated products according to a certain criteria [7,8,23]. These methods are broadly characterized as weight based sensitivity measures. Values stored in the static matrix of weights of the neuron connections in a neural network can be used to determine the relative influence of each input parameter on the network response. On the other hand, using

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the derivability property of meta-models, derivative based sensitivity measures can also be determined. In [12] a partial derivative based method using a single hidden layer neural network for determining sensitivity measures is presented.

The problem with the above mentioned methods is that they require a large number of sample points for calculating sensitivity measures and thus are computationally expensive. Also, application of these methods using meta-models requires a large number of sample points for an accurate approximation of the model response which in turn influence the sensitivity measures [13]. In this paper we introduce a new class of meta-model based global sensitivity measures termed as classification based sensitivity measures. The difference between classification based methods and previously presented methods is the granularity of the approximation. The new approach does not requires full approximation of the model response but only the level sets of the model response which in turn requires relatively lesser sample points for qualitatively good sensitivity measures. These level sets are identified by partitioning the values of the model response into a set of disjoint classes with the help of Support Vector Machines [2,24] and then calculating the influence of a model parameter on the model response with the help of change of class on that parameter domain through Monte Carlo simulation.

The next section introduces methods of global sensitivity analysis for non-linear models and highlights some meta-models particularly the Support Vector Machines and their use for the classification of the values of the model response. Section 3 explains the new classification based sensitivity measures in detail and defines new Vertical Class Jump Method, Horizontal Class Jump Method, and Boundary Method. Section 4 shows the results of these methods applied on industry relevant example problems and their comparison with existing global sensitivity methods. The conclusions are presented in Section 5.

2. Global sensitivity analysis and meta-models

As compared to the local approach for sensitivity analysis in which the effect of the variation of a parameter on a model response is computed when all others are kept constant at the nominal value, global methods evaluate the effect of a parameter while all others are varying as well. A global method should cope with the influence of scale and shape, i.e., the effect of the range of input variation and the form of its probability density function. Most general methods (non-meta-model based methods) for global sensitivity analysis, which are presented in this section, work independent of the additivity or linearity of the model and deals also with interaction effects, especially important for non-linear, non-additive models. These interaction effects arise when the effect of changing at least two factors is different from the sum of their individual effects. The interaction effects can be calculated directly using a model response or using meta-models which act as surrogate models. For information regarding meta-models such as Artificial Neural Networks (ANN) and Radial Basis Function Networks (RBFN), the reader is referred to [3,11,15]. In the following two sub-sections, commonly used general methods for global sensitivity analysis are presented. ANN specific methods known as weight based methods are presented in Section 2.3. At the end, Support Vector Machines (SVM) are explained which are import tools for calculating classification based sensitivity measures.

2.1. Variance based methods

Variance based measures are the most commonly used measures for quantifying the sensitivity of linear as well as non-linear systems [18]. Advancement in computing power has facilitated the use of variance based methods that can accommodate non-linearity and interactions within a model and its parameters. These methods are usually model independent so they can be used on models whose algorithms are complex or are not understood deterministically [16]. One such method is the variance based decomposition [19]. A response $Y = f(\underline{X})$ with $\underline{X} = (X_1, X_2, \ldots, X_n)^T$ can be represented as

$$f(\underline{X}) = f_o + \sum_{i=1}^{n} f_i(X_i) + \sum_{i=1}^{n}\sum_{j>i}^{n} f_{ij}(X_i, X_j) + \cdots + f_{12\ldots n}(X_1, X_2 \ldots, X_n). \quad (2)$$

The function $Y = f(\underline{X})$ is characterized by its variance $V(Y)$ which can be decomposed into partial variances $V_i, V_{ij}, \ldots V_{12\ldots n}$ associated with the function terms $f_i(\cdot), f_{ij}(\cdot), \ldots, f_{12\ldots n}(\cdot)$ according to Eq. (2) as

$$V(Y) = \sum_{i=1}^{n} V_i + \sum_{i=1}^{n}\sum_{j>1}^{n} V_{ij} + \cdots + V_{12\ldots n}. \quad (3)$$

The sensitivity measures $S_i^*, S_{ij}^*, \ldots, S_{12\ldots n}^*$ associated with the function terms $f_i(\cdot), f_{ij}(\cdot), \ldots, f_{12\ldots n}(\cdot)$ are calculated by dividing the variances $V_i, V_{ij}, \ldots V_{12\ldots n}$ by the total variance $V(Y)$. This identifies the contribution of the individual input parameters and the combinations of the input parameters on the total variance $V(Y)$ of the response variable Y. Eq. (4) thus follows from Eq. (3) as

$$1 = \sum_{i=1}^{n} S_i^* + \sum_{i=1}^{n}\sum_{j>1}^{n} S_{ij}^* + \cdots + S_{12\ldots n}^*. \quad (4)$$

The values S_i^* are taken as the main effects and the values $S_{ij}^*, \ldots, S_{12\ldots n}^*$ as interaction effects. In order to evaluate the total effect of a single parameter X_i, all partial sensitivity measures $S_i^*, S_{ij}^*, \ldots, S_{12\ldots n}^*$ involving X_i are summed up to define the sensitivity measure $\widetilde{S}_i$. This sensitivity measure considers the interactions among all model parameters. In order to quantify which amount of variance $V(Y)$ is caused due to a single parameter X_i, the corresponding sensitivity measures $\widetilde{S}_i$ can be normalized according to Eq. (1). The sensitivity measure $\widetilde{S}_i$ can be numerically computed using the Sobol approach [19], known as Sobol Indices, which use the Monte Carlo simulation. Sobol Indices can also be measured using a meta-model, i.e., a full approximation of the model response.

2.2. Derivative based methods

Another common method is the derivative based sensitivity method. The local sensitivity of a function $f(\underline{X})$ with $\underline{X} = (X_1, X_2, \ldots, X_n)^T$ at a certain point can be represented by the partial derivatives. In order to calculate the global sensitivity measures, the partial derivatives can be integrated over the complete input space H^n. In order to address the cancellation problem due to the change in sign, the absolute value of the derivative is taken [4].

$$g_i(\cdot) = \left|\frac{\partial f(\underline{X})}{\partial X_i}\right|. \quad (5)$$

A similar approach

$$g_i(\cdot) = \left(\frac{\partial f(\underline{X})}{\partial X_i}\right)^2 \quad (6)$$

is used in [20] as a derivative based global sensitivity measure. Thus, the global sensitivity measure $\widetilde{S}_i$ can be derived as

$$\widetilde{S}_i = \int_{H^n} g_i(\cdot), \quad (7)$$

as an estimator for the influence of $X_{i,\ i=1,2,\ldots,n}$ on a model function $f(\underline{X})$. By using Monte Carlo sampling methods Eq. (7) can be approximated as

$$\widetilde{S}_i \approx \frac{1}{N}\sum_{j=1}^{N} g_i(\underline{X}_j), \quad (8)$$