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On the sensitivity analysis of porous material models

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ABSTRACT

Porous materials are used in many vibroacoustic applications. Different available models describe their behaviors according to materials' intrinsic characteristics. For instance, in the case of porous material with rigid frame, and according to the Champoux–Allard model, five parameters are employed. In this paper, an investigation about this model sensitivity to parameters according to frequency is conducted. Sobol and FAST algorithms are used for sensitivity analysis. A strong parametric frequency dependent hierarchy is shown. Sensitivity investigations confirm that resistivity is the most influent parameter when acoustic absorption and surface impedance of porous materials with rigid frame are considered. The analysis is first performed on a wide category of porous materials, and then restricted to a polyurethane foam analysis in order to illustrate the impact of the reduction of the design space. In a second part, a sensitivity analysis is performed using the Biot–Allard model with nine parameters including mechanical effects of the frame and conclusions are drawn through numerical simulations.

1. Introduction

Porous materials are used in a variety of acoustic applications. Prediction tools of acoustic characteristics for these materials are necessary. Zwikker and Kosten [1]and Biot [2,3] developed the first popular porous media models. A thorough review of these models and further developments was performed by Attenborough [4]. For the case of rigid frame porous media, Allard [5,6] developed a five parameters model, based on the idea of Johnson [7,8]. Measurement and identification of characteristics can be difficult and time consuming, and understanding the model sensitivity can make the optimization of the sound packages easier and facilitate new concept developments. Only very few papers in the open literature on porous media deal with sensitivity of models. Some elements about first-order estimation of impact of parameters are presented for instance in [9]. This gives a useful information, which remains limited to very small variations of parameters without considering any coupling effect between them. To the author's knowledge, one of the most advanced studies on the sensitivity analysis of porous materials models has been proposed by Bolton et al. [10,11], in which a singular value decomposition is performed on the so-called sensitivity matrix, which is build from first-order estimation of derivatives (finite differences) and concatenates effects on absorption coefficient or transmission loss factor for different frequencies. A Singular Value Decomposition is then performed to check the coupling effects between the parameters (the parameters being considered as independent) in order to reduce the size of the design space for identification purpose. The aim of this contribution is to go one step further, by applying rigorous sensitivity analysis

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techniques to porous material models. For illustration purpose, the main features of interest are the acoustic impedance and the absorption coefficient of a sample of porous material backed by an impervious rigid wall. The model used for the description of the acoustic performances is the Champoux–Allard one (depending on five parameters: porosity, flow resistivity, tortuosity, viscous and thermal characteristic lengths). It should be noticed that the methodology is general and can be applied to more complicated porous material models. For instance, the considered sensitivity approach is also applied in this paper using the Biot–Allard poroelastic model.

In this paper we focus on global sensitivity analysis techniques. We classically distinguish two families of methods, namely the local and the global ones. Local sensitivity techniques are low cost, very easy to implement, but they are only able to capture the sensitivity of the model in a limited subset of the design space. On the other hand, global sensitivity analyses, which require a larger computational cost, give information about sensitivity which are valid for the whole design space and can deal with interactions effects between parameters. Sobol and FAST global sensitivity methods are considered here. The main issue is to clarify how the variability associated with the model inputs affects the model outputs [12]. Sensitivity analysis is also expected to (but not limited to) determine which input parameters contribute the most to output variabilities [13]; which parameters are insignificant; and estimate parameter interactions.

A comprehensive review of the different sensitivity analysis methods, including their advantages and drawbacks, has been proposed by Helton et al. [14] and Frey et al. [15]. A comparison of these methods can be found in [16–20]. Among the available sensitivity analysis methods, we propose in this paper to apply Sobol [21] and FAST [22].

The paper is structured as follows. Section 2 provides a brief survey of the sensitivity analysis methods considered in the paper. Section 3 recalls the porous material models used in this work. In Section 4, the Champoux–Allard model is first considered. Sensitivity results are presented and discussed. This analysis is performed considering a large design space whose parameters represent a wide variety of porous materials. Some remarks are drawn about agreements between Sobol and FAST results. Then, a comparison is performed between the first-order sensitivity and total sensitivity indexes, in order to evaluate interaction effects vs. total ones. In Section 5, another sensitivity analysis is performed when the design space is limited to a specific porous material, namely a polyurethane foam. In particular, a focus is made on the choice of the probability functions used in the sensitivity analysis. Some comments related to modelling and characterization of porous materials are given. Sections 6 and 7 are dedicated to sensitivity analysis of materials that exhibit fluid–structure coupled behavior in the frequency range of interest. To that end, the Biot–Allard model is considered, first with a large porous materials data base, then a restriction to a given type of material. Finally, some recommendations and concluding remarks are given in Section 8.

2. Sensitivity analysis methods

Most engineering problems use parametric models. Sensitivity analysis provides tools to understand the impact of each parameter on the outputs of interest. Associated methods are classically divided into two categories, namely the local sensitivity analysis techniques and the global ones. The local methods estimate the sensitivity of a given model to input parameters using different orders of partial derivatives. This type of analysis is limited to small variations of parameters and is not able to capture the coupling effects between them. The most popular technique in that category is the One-At-a-Time (OAT) technique, which basically evaluates the sensitivity of a feature f to a parameter x using the partial derivative $\partial f/\partial x$ which is estimated through finite differences $\Delta f/\Delta x$. This kind of technique is widely used and generally gives pertinent information in a local point of view.

Global sensitivity analysis [12] is required when large design space and/or coupling effects are concerned. Among possible methods the “importance measure method” or “correlation ratio technique” are capable of estimating the contribution of each parameter to the output variance [23]. However, whether a parameter is influential or not depends also on the interactions and influences of all the parameters. In this paper, two global methods are considered, namely the Fourier Amplitude Sensitivity Test (FAST) and Sobol methods, which not only can measure the “main effect” (also named first-order term) but can also compute the so-called “Total Sensitivity Indexes” (TSI) [12]. The brief outline of global sensitivity estimation is largely inspired from different references and cookbooks offered by Saltelli and Tarantola [24].

2.1. Sobol—variance analysis

Indeed, concerning a quantity of interest (the output of the model), the Total Sensitivity Index of parameter i, denoted by $\mathrm{TSI}(i)$, is defined as the sum of all the sensitivity indexes (including all the interactions effects) involving parameter i. For example, suppose that we only have three input parameters (A, B and C) in our model. The total effect of parameter A on the output is $\mathrm{TSI}(A)=\mathrm{SI}(A)+\mathrm{SI}(AB)+\mathrm{SI}(AC)+\mathrm{SI}(ABC)$. Here, $\mathrm{SI}(A)$ denotes the first-order sensitivity index for parameter A, $\mathrm{SI}(AX)$ is the second-order sensitivity index for the parameters A and X (for $X\neq A$), i.e. the interaction between parameters A and X, and so on. The first-order sensitivity index does not take into account coupling effects between parameters, but considers variation of the parameter according to its statistical distribution on a possibly large range, in that sense it is more general than the classical OAT finite differences based index.

In general, a mathematical model denoted $f(\cdot)$ is a plant connecting a set of n input parameters to an output y, namely $y=f(x)$, where x is a random vector of input parameters, characterized by a joint probability density function