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Journal of Terramechanics 49 (2012) 315–324

Journal of Terramechanics

www.elsevier.com/locate/jterra

Sensitivity analysis, calibration and validation of a snow indentation model

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Received 12 September 2012; received in revised form 23 November 2012; accepted 4 December 2012

Abstract

Quantification of the mechanical behavior of snow in response to loading is of importance in vehicle-terrain interaction studies. Snow, like other engineering materials, may be studied using indentation tests. However, unlike engineered materials with targeted and repeatable material properties, snow is a naturally-occurring, heterogeneous material whose mechanical properties display a statistical distribution. This study accounts for the statistical nature of snow behavior that is calculated from the pressure-sinkage curves from indentation tests. Recent developments in the field of statistics were used in conjunction with experimental results to calibrate, validate, and study the sensitivity of the plasticity-based snow indentation model. It was found that for material properties, in the semi-infinite zone of indentation, the cohesion has the largest influence on indentation pressure, followed by one of the the hardening coefficients. In the finite depth zone, the friction angle has the largest influence on the indentation pressure. A Bayesian metamodel was developed, and model parameters were calibrated by maximizing a Gaussian likelihood function. The calibrated model was validated using three local and global confidence-interval based metrics with good results.

Keywords: Indentation; Snow; Drucker-Prager; Validation; Calibration; Sensitivity; Bayesian; Metrics; Bias; Surrogate

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1. Introduction

A key component in the modeling of vehicle-terrain interactions for soils and snow [1–3] is the pressure-sinkage relationship of terrain material obtained using indentation testing. Naturally occurring terrain materials have been categorized as random heterogeneous materials [4] such that their properties should be treated statistically. Recent efforts in statistical modeling of vehicle-snow interaction include the interval analysis approach in [5], a metamodeling approach in [6], and a polynomial chaos approach in [7]. The pressure-sinkage relationship used in these efforts were, however, empirically-based.

Recently, a snow indentation model in [8] was developed based on plasticity theory such that pressure-sinkage curves have a physical basis, which gives an improvement over the empirical nature of previous research work. The model uses only a few fundamental material properties such as the cohesion, friction angle and hardening parameters. However, due to limited test data, the material properties of the model were assumed in an *ad hoc* fashion without consideration of the statistical variations of the pressure-sinkage curves of natural snow. Estimating parameters of a physical model against test data is a statistical process that is usually difficult since it belongs to the class of inverse problems. Indeed, estimating mechanical properties from indentation tests for engineering materials in general poses as an inverse problem.

Parameter estimation is also called calibration and is intimately related to the validation of the model. Validation of engineering and scientific models has drawn much attention from academia and industry in recent years resulting in common terminology (e.g., [9]) for validation and related statistical frameworks [10,11]. One accepted

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http://dx.doi.org/10.1016/j.jterra.2012.12.001

definition of validation is ‘the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model’ [9]. Characterization of uncertainties of model and data is an integral part of the validation process. Toward this end, advancement of flexible and reasonably rigorous statistical frameworks such as [10,11] has been made to address the issues of sensitivity, calibration of parameters and validation of models including many applications to road-load interaction in automotive engineering [11–13]. In addition, quantitative validation metrics have been an active research area that provide a more rigorous statistical assessment of the agreement between test results and model predictions [14]. Although headway has been made in the statistical modeling of vehicle-snow interaction, no statistically rigorous efforts have been made to validate the various models developed recently.

This paper applies recent results in the field of statistics, to study the sensitivity of the snow indentation model, to calibrate fundamental material properties of the model using newly obtained experimental results, and to validate the model using calibrated material properties and several validation metrics.

The paper is organized as follows. Section 2 discusses background in the snow indentation model, statistical methodology in global sensitivity analysis, Bayesian metamodel, calibration, and confidence-interval based validation metrics. Section 3 presents new snow indentation tests. Section 4 discusses results, and Section 5 follows with discussion and conclusions.

2. Background and methodology

In the following, we first summarize the snow indentation model where material constants are to be calibrated against test data toward the validation of the model. We then discuss the statistical methods and models used. It should be noted that there are two types of model discussed in this paper, one is the physical snow indentation model, and the other is the statistical model. Unless expressed explicitly, ‘model’ by itself means the snow indentation model.

2.1. Snow indentation model

Three deformation zones have been approximately identified in [8]. Zone I is a small, initially linearly elastic region. It is followed by zone II, a strain-hardening region where the pressure bulb developed underneath the indenter has not yet reached the bottom of the snow cover, i.e., it is a zone of semi-infinite depth. Zone III, a finite-depth zone, is a region where the pressure bulb has reached the bottom of the snow cover.

The material properties for the plasticity indentation model to be calibrated in this paper are summarized below. Full details of the model can be found in [8] and references therein.

A simple Drucker-Prager yield function was used to develop the plasticity solutions for the indentation model using two material parameters: the cohesion (p_d) and the friction angle (β) which can be related to the absolute tensile and compressive strengths (T and C) as

$$C - \frac{1}{3} C \tan\beta - p_d = 0 \tag{1}$$

$$T + \frac{1}{3} T \tan\beta - p_d = 0 \tag{2}$$

The hardening of the snow can be expressed by the location of the cap of the Drucker-Prager model, p_a, which is parameterized as

$$\log_{10} p_a = c_1 - c_2 \exp\left(-\epsilon_v^p - c_3\left(\epsilon_v^p\right)^3\right) \tag{3}$$

where $\epsilon_v^p = \frac{\epsilon_{kk}^p}{3}$ is the volumetric plastic strain, c_1, c_2 and c_3 are constants.

2.2. Statistical methods

In this section, we present background of the essential ingredients of the methodology used in this paper: global sensitivity analysis, Bayesian metamodel, calibration, and validation. These include the Gaussian maximum likelihood used in calibration, Gaussian processes used for metamodel and sensitivity analysis, and validation metrics. The methods used are also summarized as flow charts in Figs. 2–4 which will be discussed in detail in Section 4.

2.2.1. Gaussian likelihood

Maximum likelihood estimation (MLE) is a common method that estimates the parameters of a statistical model such that there is a maximum probability of the observed data based on the estimated parameters [15]. When the statistical model uses a Gaussian distribution for the random variable x with unknown mean μ and standard deviation σ, one has

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{4}$$

To estimate the mean and standard deviation of a sample size of N ($x_i, i = 1, \ldots, N$) using MLE, the probability of the data set is first expressed as the product of the probabilities of each point, i.e., the likelihood function $L(\mu, \sigma|x)$

$$L(\mu,\sigma|x) = \prod_{i=1}^{N} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right) \tag{5}$$

The unknown parameters (μ and σ) are then obtained by maximizing L which is equivalent in maximizing $\log(L)$ such that the normal log-likelihood function is

$$\log(L) = -\frac{N}{2}\left(\log\sigma^2 + \log(2\pi)\right) - \sum_{i=1}^{N}\left(\frac{(x_i-\mu)^2}{2\sigma^2}\right) \tag{6}$$