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Parameter identification of multibody systems based on second order sensitivity analysis ☆

Jieyu Ding [a,*], Zhenkuan Pan [a], Liqun Chen [b]

[a] College of Information Engineering, Qingdao University, Qingdao 266071, China
[b] Department of Mechanics, Shanghai University, Shanghai 200444, China

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ABSTRACT

An identification procedure for multibody systems is presented to determine optimal values for unknown or estimated model parameters from experimental test data. Based on a non-linear least-square optimization procedure, the Levenberg–Marquardt trust region method is developed to estimate the unknown parameters, in which the second order sensitivity analysis with the hybrid method is applied to improve the convergence and stability. Finally, numerical examples of slider-crank mechanism validate the accuracy and efficiency of the method presented.

1. Introduction

Parameter identification of multibody systems is to determine the geometric parameters and inertia parameters according to observed responses of the system, which can be applied in diagnostic or testing procedures to a given system or building the motion equations of systems via inverse modeling methods. The objective functions are often established through the least-square method and a typical optimization problem including variables depending on parameters must be solved.

Many optimization methods for estimating unknown parameters in non-linear dynamical systems have been developed. Schiehlen and Hu [1] use correlation techniques to overcome the disadvantage of the least-square method, which yields often biased estimates. Sujan and Dubowsky [2] present a mutual-information-based observability metric for the online dynamic parameter identification of a multibody system. Grotjahn et al. [3] present an approach for the identification of the dynamics of complex parallel mechanisms, which separates friction and rigid-body dynamics using simple PTP motions. Hardeman et al. [4] describe a finite element based approach for the automatic generation of models suitable for dynamic parameter identification. Bauchau and Wang [5] implement a system identification algorithm, which uniquely combines the methods of minimum realization and subspace identification. Kim et al. [6] identify the parameters of several typical damping models for multibody dynamic simulation. Serban and Freeman [7] use Levenberg–Marquardt methods to solve the non-linear least-squares problem, which need partial derivative computed through sensitivity analysis. But only the first order derivative information is considered in their studies while second order sensitivity analysis of the constrained multibody system model is developed recently [8,9].

In this paper, an approach of parameter identification for multibody systems based on second order sensitivity analysis is presented. A Levenberg–Marquardt trust region method is used to estimate the unknown parameters, in which the second order sensitivity analysis is applied to improve the convergence and stability.

2. Problem formulation

Multibody systems under consideration can be described by the following differential-algebraic equations of motion:

$$\begin{cases} \boldsymbol{M}(\boldsymbol{q},\boldsymbol{b})\ddot{\boldsymbol{q}}+\boldsymbol{\Phi}_{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{\lambda}=\boldsymbol{Q}(\dot{\boldsymbol{q}},\boldsymbol{q},\boldsymbol{b},t) & (1.1)\\ \boldsymbol{\Phi}(\boldsymbol{q},\boldsymbol{b},t)=\boldsymbol{0} & (1.2)\end{cases}$$

where $\boldsymbol{b}\in R^p$ is a vector of design variables, $\boldsymbol{q}=\boldsymbol{q}(t)\in R^n$ is a vector of generalized coordinates that describes dynamic response of the system. The constraints in algebraic equations (1.2) lead to reaction forces expressed in the differential equations (1.1) by Lagrange multipliers $\boldsymbol{\lambda}\in R^m$ and the Jacobian matrix $\boldsymbol{\Phi}_{\boldsymbol{q}}\in R^{m\times n}$. The holonomic constraint functions $\boldsymbol{\Phi}\in R^m$, the generalized mass matrix $\boldsymbol{M}\in R^{n\times n}$, and the applied forces vector $\boldsymbol{Q}\in R^n$ depend on the design variables.

A complete characterization of the motion of the system requires definition of initial conditions on position $\boldsymbol{\varphi}\in R^{n-m}$ and

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* Corresponding author.
E-mail addresses: djy@qdu.edu.cn (J. Ding), zkpan@qdu.edu.cn (Z. Pan), lqchen@staff.shu.edu.cn (L. Chen).

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velocity $\overline{\varphi} \in R^{n-m}$ in the following form:

$$\varphi(\boldsymbol{q}^1,\boldsymbol{b},t^1)=\boldsymbol{0} \tag{2.1}$$

$$\overline{\varphi}(\dot{\boldsymbol{q}}^1,\boldsymbol{q}^1,\boldsymbol{b},t^1)=\boldsymbol{0} \tag{2.2}$$

where $t^1$is initial time and the matrices $\begin{pmatrix} \boldsymbol{\Phi}_{\boldsymbol{q}}(t^1) \\ \boldsymbol{\varphi}_{\boldsymbol{q}^1} \end{pmatrix}_{n\times n}$ and $\begin{pmatrix} \boldsymbol{\Phi}_{\boldsymbol{q}}(t^1) \\ \overline{\boldsymbol{\varphi}}_{\dot{\boldsymbol{q}}^1} \end{pmatrix}_{n\times n}$ must be non-singular.

The typical objective function is established by differences between the observed datay_i $(i=1,\ldots,n_m)$ and the responses $y_m(t_i)(i=1,\ldots,n_m)$ from dynamic equations of multibody systems, which can be noted as

$$\psi(\boldsymbol{b})=\frac{1}{2}\sum_{i=1}^{n_m}(y_i-y_m(t_i))^2 \tag{3}$$

The number of observed data points $n_m$is generally far more than the number of parameters p. Since any of the terms in Eqs. (1) can be functions of the unknown parameters $\boldsymbol{b}$, $y_m(t)$is of the form

$$y_m(t)=f(\ddot{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{q},\boldsymbol{\lambda},\boldsymbol{b},t) \tag{4}$$

3. Optimization procedure

The estimate of the design parameters $\boldsymbol{b}$, based on n_m data points, can be found through the minimization procedure of Eq. (3). If there are some constraints in optimization problem, the augmented Lagrange method (ALM), which combines the Lagrange function with the penalty function of sequential unconstrained minimization technique (SUMT), can be used to translate constrained optimization problem into unconstrained optimization problem. As the procedure of ALM has been presented in [10], only unconstrained optimization procedure is considered in this paper. To improve the convergence and stability, the Levenberg–Marquardt trust region method is used to estimate the unknown parameters.

Around the current search point $\boldsymbol{b}^{(k)}$, the trust region method has a region

$$\Omega^{(k)}=\{\boldsymbol{b}\,|\,\|\boldsymbol{b}-\boldsymbol{b}^{(k)}\|\le h^{(k)}\} \tag{5}$$

where the quadratic model

$$\begin{aligned}&\min_{\boldsymbol{b}\in R^n}\left\{\varphi^{(k)}(\boldsymbol{d})=\psi(\boldsymbol{b}^{(k)})+(\boldsymbol{g}^{(k)})^{\mathrm{T}}\boldsymbol{d}+\tfrac{1}{2}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{G}_{(k)}\boldsymbol{d}\right\}\\ &\text{s.t.}\|\boldsymbol{d}\|\le h^{(k)}\end{aligned} \tag{6}$$

is trusted to be correct and steps are chosen to stay within this region. $\boldsymbol{g}^{(k)}$ and $\boldsymbol{G}^{(k)}$ are defined as the first derivative and the second derivative of Eq. (3) at the current search point $\boldsymbol{b}^{(k)}$, respectively.

The size of the region $\Omega^{(k)}$ is modified during the search, based on how well the model agrees with actual function evaluations. The following ratio of actual to predicted reduction can be used to measure the approximation:

$$r^{(k)}=\frac{\Delta\psi^{(k)}}{\Delta\varphi^{(k)}}=\frac{\psi(\boldsymbol{b}^{(k)})-\psi(\boldsymbol{b}^{(k)}+\boldsymbol{d}^{(k)})}{\varphi^{(k)}(\boldsymbol{0})-\varphi^{(k)}(\boldsymbol{d}^{(k)})} \tag{7}$$

where the step $\boldsymbol{d}^{(k)}$ is the solution of model (6). If $r^{(k)}$is close to 1, then the quadratic model is quite a good predictor and the region can be increased in size. On the other hand, if $r^{(k)}$is too small, the region is decreased in size.

The Levenberg–Marquardt trust region method [11,12] with second order sensitivity analysis can be described as the following form:

Step 1: Given design parameters $\boldsymbol{b}^{(1)}$, and $\sigma^{(1)}>0,\varepsilon>0,1>c_2>c_1\ge 0,k=1$.
Step 2: Use the first order and the second order sensitivity analysis methods to get and $\boldsymbol{G}^{(k)}$. If $\boldsymbol{G}^{(k)}+\sigma^{(k)}\boldsymbol{I}$ is positive definite, then go to step 3. Else, set $\sigma^{(k)}=4\sigma^{(k)}$, repeat this step until $\boldsymbol{G}^{(k)}+\sigma^{(k)}\boldsymbol{I}$ is positive definite.
Step 3: Compute $\boldsymbol{d}^{(k)}=-[\boldsymbol{G}^{(k)}+\sigma^{(k)}\boldsymbol{I}]^{-1}\boldsymbol{g}^{(k)}$, if $\|\boldsymbol{d}^{(k)}\|\le\varepsilon$, then stop and the best solution is $\boldsymbol{b}=\boldsymbol{b}^{(k)}$. Else, go to step 4.
Step 4: Compute $r^{(k)}$through Eq. (7). Chose $\sigma^{(k+1)}$ as

$$\sigma^{(k+1)}=\begin{cases}4\sigma^{(k)} & r^{(k)}<c_1\\ \frac{1}{2}\sigma^{(k)} & r^{(k)}>c_2\\ \sigma^{(k)} & \text{otherwise}\end{cases} \tag{8}$$

If $r^{(k)}\le 0$, set $\boldsymbol{b}^{(k+1)}=\boldsymbol{b}^{(k)}$, $k=k+1$, go to step 3. Else, set $\boldsymbol{b}^{(k+1)}=\boldsymbol{b}^{(k)}+\boldsymbol{d}^{(k)}$, $k=k+1$, go to step 2.

4. Sensitivity analysis

The first derivative and the second derivative of Eq. (3) are

$$\nabla\psi(\boldsymbol{b})=-\sum_{i=1}^{n_m}[(y_i-y_m(t_i))\cdot\nabla y_m(t_i)] \tag{9}$$

$$\nabla^2\psi(\boldsymbol{b})=\sum_{i=1}^{n_m}(\nabla y_m(t_i))^T\nabla y_m(t_i)-\sum_{i=1}^{n_m}[(y_i-y_m(t_i))\cdot\nabla^2 y_m(t_i)] \tag{10}$$

where

$$\nabla y_m(t)=f_{\ddot{\boldsymbol{q}}}\ddot{\boldsymbol{q}}_{\boldsymbol{b}}+f_{\dot{\boldsymbol{q}}}\dot{\boldsymbol{q}}_{\boldsymbol{b}}+f_{\boldsymbol{q}}\boldsymbol{q}_{\boldsymbol{b}}+f_{\lambda}\boldsymbol{\lambda}_{\boldsymbol{b}}+f_{\boldsymbol{b}}\triangleq Df \tag{11}$$

$$\begin{aligned}\nabla^2 y_m(t)&=f_{\ddot{\boldsymbol{q}}}\ddot{\boldsymbol{q}}_{\mathrm{bb}}+f_{\dot{\boldsymbol{q}}}\dot{\boldsymbol{q}}_{\mathrm{bb}}+f_{\boldsymbol{q}}\boldsymbol{q}_{\mathrm{bb}}+f_{\lambda}\boldsymbol{\lambda}_{\mathrm{bb}}+Df_{,\ddot{\boldsymbol{q}}}\ddot{\boldsymbol{q}}_{\boldsymbol{b}}\\&\quad+Df_{,\dot{\boldsymbol{q}}}\dot{\boldsymbol{q}}_{\boldsymbol{b}}+Df_{,\boldsymbol{q}}\boldsymbol{q}_{\boldsymbol{b}}+Df_{,\lambda}\boldsymbol{\lambda}_{\boldsymbol{b}}+Df_{,\boldsymbol{b}}\\&\triangleq f_{\ddot{\boldsymbol{q}}}\ddot{\boldsymbol{q}}_{\mathrm{bb}}+f_{\dot{\boldsymbol{q}}}\dot{\boldsymbol{q}}_{\mathrm{bb}}+f_{\boldsymbol{q}}\boldsymbol{q}_{\mathrm{bb}}+f_{\lambda}\boldsymbol{\lambda}_{\mathrm{bb}}+Df_{\boldsymbol{b}}\end{aligned} \tag{12}$$

Thus sensitivities of the generalized coordinates, of the generalized coordinate derivatives, and of the Lagrange multipliers with respect to the design parameters are required.

Direct differentiation method and adjoint variable method are two major analytical techniques for sensitivity analysis. Previous study shows that for large numbers of design variables, adjoint variable method is more efficient in the first order sensitivity analysis, while the hybrid method is superior in the second order sensitivity analysis [8,9].

Consider the objective function $\psi(\boldsymbol{b})$ as the discrete form on time interval $[t_1,t_{n_m}]\triangleq[t^1,t^2]$, the first derivative of $\psi(\boldsymbol{b})$ with respect to design variables can be evaluated by direct differentiation method, and the second derivative can be evaluated by the hybrid approach.

4.1. First order sensitivity analysis

The derivative of Eq. (1.1) with respect to design variables is

$$\boldsymbol{M}\ddot{\boldsymbol{q}}_{\boldsymbol{b}}-\boldsymbol{Q}_{\dot{\boldsymbol{q}}}\dot{\boldsymbol{q}}_{\boldsymbol{b}}+\tilde{\boldsymbol{\Pi}}_{\boldsymbol{q}}\boldsymbol{q}_{\boldsymbol{b}}+\boldsymbol{\Phi}_{\boldsymbol{q}}^{\mathrm{T}}\lambda_{\boldsymbol{b}}+\tilde{\boldsymbol{\Pi}}_{\boldsymbol{b}}=\boldsymbol{0} \tag{13}$$

where

$$\tilde{\boldsymbol{\Pi}}\triangleq M\tilde{\ddot{\boldsymbol{q}}}+\boldsymbol{\Phi}_{\boldsymbol{q}}^{\mathrm{T}}\tilde{\lambda}-\boldsymbol{Q} \tag{14}$$

The symbol "$\sim$" above a variable denotes that the variable is to be held fixed for the partial differentiation indicated.