



A study on design sensitivity analysis for general nonlinear eigenproblems

Li Li, Yujin Hu*, Xuelin Wang

School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China

ARTICLE INFO

Article history:

Received 11 May 2012

Received in revised form

20 July 2012

Accepted 4 August 2012

Available online 29 August 2012

Keywords:

Sensitivity analysis

Viscoelastic structure

Eigenvector sensitivity

Repeated eigenvalues

Nonlinear eigenproblem

Nonviscous damping

ABSTRACT

A general nonlinear eigenproblem is considered in this paper. It is shown that the widely used undamped, viscously or nonviscously damped eigenproblem can be considered as a special case of the more general nonlinear eigenproblem. The existing formulas of the derivatives of eigensolutions of this nonlinear eigenproblem are very complex since the normalizations of undamped eigenproblems are considered in their studies. To simplify the computation of eigensensitivity, a new normalization of the general nonlinear eigenproblem is presented. The new normalization of the general eigenproblem derived here can degenerate to the familiar mass orthogonal relationship of undamped eigenvectors. Based on this normalization, the design sensitivity analysis for the general nonlinear eigenproblem with respect to arbitrary design parameters is studied. Moreover, this method can address the eigenproblem for both repeated and distinct eigenvalues. As it can be seen, under such general normalization the derivatives of eigensolutions can be expressed in a way similar to those of undamped systems. Finally, five numerical examples are provided to illustrate the effectiveness of the derived results. It is shown that the derivatives of eigensolutions can be treated in a unified way for different structural systems (i.e., undamped systems, viscously and nonviscously damped systems, and nonlinear systems).

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

Design sensitivity analysis of eigenproblems of structural and mechanical systems with respect to structural design parameters has become an integral part of many engineering design methodologies including structural health monitoring, structural reliability, dynamic model updating, structural design optimization, structural dynamic modification, approximate reanalysis techniques and many other applications. Mottershead et al. [1] pointed out that the eigensolution sensitivity method is probably the most successful of the many approaches to the problem of models updating and has developed into a mature technology applied successfully for the correction of enterprise-level finite element models. Although computing eigenvalue sensitivity is straightforward, finding eigenvector derivatives resides several challenges. There are two main difficulties residing in computing the eigenvector derivatives. One of the main difficulties in computation of eigenvector sensitivities is the singularity issue. And the other difficulty is the derivatives of eigenvectors corresponding to the repeated eigenvalues. Eigensensitivity analysis has received much attention over the past four decades. Several methods have been developed for the calculation of the eigensolution sensitivities.

* Corresponding author. Tel.: +86 27 87543972.

E-mail addresses: lili_em@126.com (L. Li), yjhu@mail.hust.edu.cn (Y. Hu), wangxl@mail.hust.edu.cn (X. Wang).

For undamped eigensystems, it is well known that the equations of motion for the free vibration can be expressed by

$$\mathbf{K}\mathbf{u}(s) = s\mathbf{M}\mathbf{u}(s) \quad (1)$$

where \mathbf{K} , $\mathbf{M} \in \mathbb{R}^{N \times N}$ are, respectively, the stiffness and mass matrices. The eigenvalues λ_i are the roots of the characteristic equation, $\det[\mathbf{K} - s\mathbf{M}] = 0$. The i th eigenvalue can be expressed as $\lambda_i = \omega_i^2$ where ω_i is the i th undamped natural frequency. Fox and Kapoor [2] derived the expressions of design sensitivity analysis of eigenproblem with respect to any design variable for symmetric undamped systems. To simplify the computation of design sensitivity of eigenproblem, Nelson [3] presented an efficient approach to calculate the eigenvector derivatives with distinct eigenvalues for undamped systems by expressing the derivative of each eigenvector as a particular solution and a homogeneous solution. One of the main advantages of Nelson's method is that it requires the information of only those eigenvectors that are to be differentiated. Sutter et al. [4] presented a comparison of several methods on calculating vibration mode shape derivatives and pointed out that Nelson's method may be more efficient than the modal method for the reason that the modal method needs all or most of the eigenvectors to determine the eigenvector derivatives. Later, Lee and Jung [5,6] proposed an efficient algebraic method with symmetric coefficient matrices for calculating the derivatives of vibration mode shapes of symmetric undamped systems with distinct and repeated eigenvalues, respectively. The main idea of this method is to obtain a non-singular equation for computing the eigensensitivity by assembling the derivatives of eigenproblems and the additional constraints of eigensolution derivatives into a linear system of algebraic equations. Unfortunately, Wu et al. [7] pointed out that the method [6] was not correct because a mistake was made in the derivation of equations on the derivatives of the normalization for the systems with repeated eigenvalues. There are many repeated eigenvalues or nearly equal eigenvalues in typical structural or mechanical systems for certain reasons. Ojalvo [8], Mills-Curran [9], Dailey [10] developed Nelson's method for solving the derivatives of eigensolutions of the real symmetric eigensystems with repeated eigenvalues. Shaw and Jayasuriya [11] generalized these methods to calculate the derivatives of eigensolutions in the case of repeated eigenvalues with repeated first-order derivatives.

Eigensensitivity analysis for viscously damped systems has received much attention over the past two decades. The symmetric eigenproblem with viscous damping is given by the second-order system

$$(s^2\mathbf{M} + s\mathbf{D}_v + \mathbf{K})\mathbf{u}(s) = \mathbf{0} \quad (2)$$

where \mathbf{D}_v is the viscous damping matrix. Adhikari [12,13] presented N -space modal methods to compute the eigenvector sensitivities for non-proportionally symmetric systems with viscous damping. Later, Adhikari and Friswell [14] extended the N -space modal method to asymmetric damped systems and the second-order eigensolution derivatives were also derived. Lee et al. [15] further extended their algebraic method to the case of distinct eigenvalues in terms of second-order symmetric viscously damped systems. Friswell and Adhikari [16] extended Nelson's method to symmetric and asymmetric systems with viscous damping. Choi et al. [17] presented an efficient algebraic method for symmetric and asymmetric viscously damped systems. One of the main advantages is that this method can compute the derivatives of eigensolutions of asymmetric systems without using the left eigenvectors. Guedria et al. [18] developed an algebraic method to calculate the derivatives of eigensolutions for general asymmetric viscously damped systems. Chouchane et al. [19] simply reviewed the algebraic method for symmetric and asymmetric systems and developed their method to the second-order derivatives of eigensolutions. Mirzaeifar et al. [20] presented a new method based on a combination of the algebraic method and the modal method for general asymmetric viscously damped systems. This combined method neither has the complications of the modal method on the computation of the complex left and right eigenvector derivatives nor suffers from the numerical instability issues usually associated with the algebraic method. Later, Xu and Wu [21] derived a new normalization and presented a new method for computing the derivatives of eigensolutions of asymmetric viscously damped eigensystems with distinct and repeated eigenvalues. Recently, Xu et al. [22] derived an efficient algebraic method for the computations of derivatives of eigensolutions of asymmetric damped eigensystems with distinct eigenvalues. More recently, Li et al. [23] suggested a new normalization for the left eigenvectors, from which the left and right eigenvector derivatives can be computed separately and independently for asymmetric eigensystems. Moreover, this method can address the eigenproblem with distinct and repeated eigenvalues and is well-conditioned since the components of coefficient matrices are all of the same order of magnitude.

These studies mentioned above only consider viscous damping model, but the viscous damping is not only the damping mechanism within the scope of linear dynamic analysis (e.g. the damping in composite materials). In principle, any causal model may be a candidate for a damping model, as long as it makes the energy dissipation functional non-negative. Increasing the use of composite structural materials, active control and damage tolerant systems in satellites, shuttles, ships, rockets, spacecrafts and automobiles has led to renewed demand for accurate and efficient analysis of nonviscous (viscoelastic) damping systems. Possibly the most general method of nonviscous damping model within the scope of linear dynamic analysis is to consider the damping forces depend on the past history of motion via convolution integrals over kernel functions, that is, this damping model should take into account the hysteretic and frequency-dependent behavior. The equations of motion describing free vibration of an N -degree-of-freedom linear system with nonviscous damping is usually expressed by [24–26]

$$\mathbf{M}\dot{\mathbf{q}}(t) + \int_{-\infty}^t \mathbf{g}(t-\tau)\dot{\mathbf{q}}(\tau)d\tau + \mathbf{K}\mathbf{q}(t) = \mathbf{0} \quad (3)$$

متن کامل مقاله

دریافت فوری ←

ISIArticles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات