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Isogeometric shape design sensitivity analysis using transformed basis functions for Kronecker delta property

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ABSTRACT

The isogeometric shape design sensitivity analysis (DSA) includes the desirable features; easy design parameterization and accurate shape sensitivity embedding the higher-order geometric information of curvature and normal vector. Due to the non-interpolatory property of NURBS basis, however, the imposition of essential boundary condition is not so straightforward in the isogeometric method. Taking advantages of geometrically exact property, an isogeometric DSA method is developed applying a mixed transformation to handle the boundary condition. A set of control point and NURBS basis function is added using the *h*-refinement and Newton iterations to precisely locate the control point to impose the boundary condition. In spite of additional transformation, its computation cost is comparable to the original one with penalty approach since the obtained Kronecker delta property enables to reduce the size of system matrix. Through demonstrative numerical examples, the effectiveness, accuracy, and computing cost of the developed DSA method are discussed.

1. Introduction

Ever since the framework of isogeometric analysis method was established by Hughes et al. [13], the isogeometric method that employs the same basis functions as used in a CAD model has shown many advantages over standard finite element methods and rapidly widened its applications such as structural vibrations [9], fluid–structure interactions [2], isogeometric sensitivity analyses enhanced by T-splines [12], turbulent flow simulations [3], and isogeometric shell analyses [4]. The isogeometric method has a major feature such as the CAD based parameterization of field variables in an isoparametric manner and thus requires no geometric parameterization during refinement processes, which are extensively discussed by Cottrell et al. [8] as the analogues of *h*-, *p*-, and *hp*-refinements in standard finite element methods. In the perspective of analysis, the non uniform rational B-spline (NURBS) basis function has desirable properties such as the partition of unity, a compact support, and non-negativity. However, it has non-interpolatory property in common with meshfree methods.

In general, the isogeometric analysis method can handle almost all sorts of essential boundary conditions as long as they can be described by the NURBS functions. However, in specific cases like pointwise essential boundary conditions that cannot be represented by a NURBS, weak enforcement or any other treatment is required. This is one of the disadvantages in isogeometric and meshfree methods due to the non-interpolating property of basis functions. Several methods have been proposed to impose essential boundary conditions for the meshfree methods. These techniques can be classified in two groups: (1) modification of the weak form; the Lagrange multiplier method, the penalty method, and the Nitsche's method, and (2) modification of shape functions summarized by Fernandez-Mendez and Huerta [11]. Both methods are applicable for the isogeometric analysis but the latter is more intuitive by applying the modified basis functions directly in the isogeometric analysis. As an application of the modification of basis functions in the isogeometric analysis, Wang and Xuan [20] used a mixed transformation method originated by Chen and Wang [5], where the complete separation of interior and boundary nodes is demonstrated, using the NURBS property that the value of interior shape functions vanishes at boundary.

In shape optimization problems, the isogeometric approach has shown two significant benefits addressed by Cho and Ha [6]; (1) the accurate sensitivity of complex geometries including the higher order effects originated from the exact representation of geometry and (2) the vast simplification of design parameterization utilizing the direct variation of CAD geometry. However, the updates of interior control points are still challenging since a CAD system typically includes only boundary information. The movement of internal control points can be related to the changes of boundary control points through an algebraic constraint [17,19]. Recently,

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Manh et al. [14] proposed an optimization scheme of the jacobian parameters, to obtain proper the control point updates for interior surface parameterization.

However, due to the lack of Kronecker delta property of NURBS basis functions, the application of the DSA method is still limited. In this paper, for a gradient-based design optimization that includes point loading and boundary conditions, a shape DSA method is developed using the mixed transformation method in the isogeometric framework. This paper is organized as follows. In Section 2, we briefly summarize the isogeometric analysis method and the treatment of essential boundary conditions. In Section 3, after a brief review of isogeometric shape DSA, the treatment of essential boundary conditions is explained. In Section 4 of numerical examples, the accuracy of isogeometric shape DSA is compared with the finite difference one for the problem having essential boundary conditions. Also, the numerical costs for the mixed transformation are discussed in the isogemetric analysis and the shape DSA. *Finally*, we draw some conclusions, which could eventually present the superior points of the proposed method in shape design optimization having essential boundary conditions.

2. Review of isogeometric method

2.1. NURBS basis function

In the isogeometric analysis, the solution space is represented in terms of the same basis functions as used to represent the geometry. The isogeometric analysis has several advantages over the conventional finite element analysis (FEA): *geometric exactness* and *simple refinements* due to the use of NURBS basis functions. Consider a knot vector ξ in one dimensional space, which is the set of coordinates ξ_i in a parametric space.

$$\xi = \{\xi_1, \xi_2, \cdots, \xi_{n+p+1}\}, \tag{1}$$

where p and n are the order and the number of basis functions, respectively. The B-spline basis functions are defined, recursively, as

$$\phi_i^0(\xi) = \left\{ \begin{array}{ll} 1 & \textit{if } \xi_i \leqslant \xi < \xi_{i+1} \\ 0 & \textit{otherwise} \end{array} \right\}, \ (p = 0), \tag{2}$$

and

$$\phi_i^p(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i}\phi_i^{p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}}\phi_{i+1}^{p-1}(\xi), (p = 1, 2, 3, ...,). \tag{3}$$

Using the B-spline basis functions and weights, the NURBS is also defined by

$$R_i^p(\xi) = \frac{\phi_i^p(\xi)w_i}{\sum_{j=0}^n \phi_j^p(\xi)w_j}, (p = 1, 2, 3, \ldots,). \tag{4}$$

Generally, the isogeometric approach using higher order basis functions offers higher regularity than conventional finite element approach. The NURBS has the following desirable properties as a basis function:

(1) $\sum_{i=1}^l R_i^p(\xi) = 1$ (Partition of unity).
(2) R_i^p is included in the interval $[\xi_i, \ \xi_{i+p+1}]$ (Compactness).
(3) $R_i^p(\xi) \geqslant 0$ (Non-negativity).

For the given l pairs of the p-th order NURBS basis function R_i^p and the corresponding (projective) control point $B_i \equiv B(x_i)$, the NURBS curve is obtained by

$$S(\xi) = \sum_{i=1}^l R_i^p(\xi)B_i. \tag{5}$$

In higher dimensional spaces, the NURBS surfaces and solids are defined, using a tensor product respectively, as

$$S(\xi, \eta) = \sum_{i=1}^l \sum_{j=1}^m R_{i,j}^{pq}(\xi, \eta)B_{i,j} \tag{6}$$

and

$$S(\xi, \eta, \zeta) = \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n R_{i,j,k}^{pqr}(\xi, \eta, \zeta)B_{i,j,k}. \tag{7}$$

For the details of NURBS geometry, interested readers may consult Rogers [18], Piegl and Tiller [15], and Farin [10]. For the brevity of expression, Eq. (7) is rewritten as

$$S(\boldsymbol{\Xi}) \equiv S(\xi, \eta, \zeta) = \sum_I W_I(\boldsymbol{\Xi})B_I(x_1, x_2, x_3). \tag{8}$$

Besides the aforementioned properties as a basis function, the constructed NURBS basis functions possess the property of affine covariance and $(p-1)$ continuous differentiability. If the knots are repeated q-times, the continuity of NURBS basis functions decreases by q as well.

2.2. Isogeometric analysis of Laplacian problems

Consider a Laplacian problem shown in Fig. 1. The body occupies an open domain Ω bounded by a closed surface Γ. The boundaries are composed of a prescribed essential boundary Γ^D and a prescribed natural boundary Γ^N, which are mutually disjointed as

$$\Gamma = \Gamma^D \cup \Gamma^N \text{ and } \Gamma^D \cap \Gamma^N = \phi. \tag{9}$$

The body is subjected to a source s and the prescribed flux t on Γ^N. $\mathbf{n}$ is an outward unit vector normal to the boundary. Using the principle of virtual work, an equilibrium equation is expressed as

$$a(\mathbf{z}, \bar{\mathbf{z}}) = \ell(\bar{\mathbf{z}}), \ \forall \bar{\mathbf{z}} \in \bar{\Theta}, \tag{10}$$

where the bilinear and linear forms are defined, respectively, as

$$a(\mathbf{z}, \bar{\mathbf{z}}) = \int_\Omega z_{,i}\bar{z}_{,i}d\Omega \tag{11}$$

and

$$\ell(\bar{\mathbf{z}}) = \int_\Omega s_i\bar{z}_i d\Omega + \int_{\Gamma^N} t_i\bar{z}_i d\Gamma. \tag{12}$$

Note that $(\bullet)_{,i}$ represents the covariant differentiation with respect to the coordinate i. Also, $\bar{\Theta}$ is a d-dimensional variational space defined by

$$\bar{\Theta} = \{ \ \bar{\mathbf{z}} \in [H^1(\Omega)]^d : \ \ \bar{\mathbf{z}} = 0 \ \text{ on } \Gamma^D \ \}. \tag{13}$$

Using isoparametric mapping, a geometric point and a response are expressed, in terms of NURBS basis functions combined with the control points $\mathbf{B}_I = \mathbf{B}(x_1, x_2, x_3)$ and with the response coefficients $\mathbf{d}_I = \mathbf{d}(x_1, x_2, x_3)$ respectively, as

$$\mathbf{x}(\boldsymbol{\Xi}) = \sum_I W_I(\boldsymbol{\Xi})\mathbf{B}_I \tag{14}$$

and

$$\mathbf{z}(\boldsymbol{\Xi}) = \sum_I W_I(\boldsymbol{\Xi})\mathbf{d}_I. \tag{15}$$

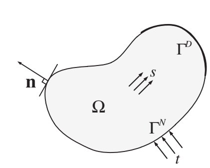

Fig. 1. Laplacian problem.