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Multi-parameter perturbation methods for the eigensolution sensitivity analysis of nearly-resonant non-defective multi-degree-of-freedom systems

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ABSTRACT

The dynamic behavior of structural systems may be strongly characterized by the occurrence of multiple internal resonances for particular combinations of the mechanical parameters. The linear models governing these resonant or nearly-resonant systems tend to exhibit high sensitivity of the eigenvalues and eigenvectors to small parameter modifications. This pathological condition is recognized as a source of relevant phenomena, such as frequency veering and mode localization or hybridization. The paper presents the generalization of uniformly valid perturbation methods to perform eigensolution sensitivity analyses in multi-degree-of-freedom Hamiltonian systems with a generic number of close eigenvalues. The leading idea is to treat systematically nearly-resonant systems as multi-parameter perturbations of a perfectly-resonant, non-defective – though a priori unknown – reference system. Given a single nearly-resonant system, a multi-parameter perturbation method is presented to achieve a two-fold objective: first, identify a close resonant system suited to serve as a starting point for sensitivity analyses (*inverse problem*); second, to approximate asymptotically the eigensolution of all the nearly-resonant systems which may arise from its generic perturbation (*direct problem*). The conditions of existence and uniqueness of the inverse problem solution are discussed. The direct problem solution is analyzed with a focus on the eigensolution sensitivity to parameter perturbations with different physical meanings, such as a slight geometric disorder or weak elastic coupling in periodic structures. Finally, the procedure is verified on a prototypical structural system describing the section dynamics of a suspended bridge.

0. Introduction

The linear dynamic models of several structural systems show that eigenvalues and eigenvectors may possess high sensitivity to the physical parameters. Since the eigenproperties strongly characterize the dynamic behavior, tracking the eigensolution loci against one or more significant parameters represents a matter of theoretical and practical interest in several fields, including stability analysis, design optimization, model updating, structural identification, and vibration control. Within the parameter space, the regions corresponding to the intersection, or closeness, between two or more eigenvalue loci are worth particular attention. Internal resonance or near-resonance conditions may activate significant

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phenomena, such as frequency crossing or veering, vibration localization and modal hybridization [1–3]. At the same time, important energy transfers among the resonant modes may characterize the system dynamic response [4].

In principle each structural system, even if realized by joining dissimilar components with strongly different stiffness and mass properties [5], may exhibit close natural frequencies for particular parameter combinations. Nonetheless, in the civil and mechanical engineering context, the near-resonance condition tends traditionally to be considered a frequency *mistuning*, and nearly-resonant systems are typically regarded as *imperfect* structures. Such a dominant idea is motivated by recurrent observations that, in the modal analysis of conservative or proportionally-damped structures, the exact coincidence of two or more frequencies mainly occurs in symmetric systems (horizontal cables, arch bridges, circular disks, cylindrical shafts, spheroidal shells, etc.), or in periodic assemblies of identical subsystems (pendulum chains, multi-span beams, planetary gears, etc.). In all these systems small frequency shifts, leading to near-resonance conditions, may appear when slight imperfections, typically geometric defects, little damages or loosening boundary constraints, destroy the nominal structural symmetry or periodicity [6–19]. Even when the structural symmetry or periodicity is preserved, similar mistuning effects can equally be originated by different kinds of weak internal symmetric or skew-symmetric interactions, introduced for instance by soft elastic links among periodic subsystems, or by piezoelectric connections in electromechanical systems [20].

In the well-established scientific literature about periodic and symmetric systems, weak linear interactions and small imperfections are respectively referred to as *coupling* and *disorder* terms, and can both be regarded as slight perturbations of a nominally perfect (*uncoupled* and *ordered*) system. In addressing the eigensolution sensitivity problem for these structures, perturbation methods may represent a valid alternative to the common, time-consuming, numerical techniques of eigensolution continuation, which are prone to fail in resolving and following a cluster of close and rapidly-evolving solutions. According to the perturbation strategy, knowing an initial reference system, the eigensolution of close *coupled* and *disordered* systems can be approximated by constructing asymptotical expansions. The traditional approach for conservative systems consists in including the disorder in the unperturbed reference system, and then performing a single-parameter analysis considering only the coupling as a perturbation [1,6]. However, such a perturbation scheme presents some drawbacks, essentially related to the small validity range of the achievable solutions. A refined approach consists instead of retaining both the disorder and the coupling as independent perturbations of the uncoupled and ordered system, and then performing a multi-parameter analysis. Among other advantages, this refinement extends the solution validity over a wider parameter range, and also allows the treatment of non-conservative discrete systems governed by nearly defective state matrices, which are the object of its original development [21].

When dealing with complex dynamic systems, characterized by several degrees of freedom, governed by a large set of mechanical parameters and exhibiting multiple internal resonances, the application of perturbation methods tends to become increasingly involved. In contrast with the simple resonant systems commonly employed as examples in the literature studies, severe difficulties or bottlenecks may occur in many technical operations, crucial for the consistency or effectiveness of the sought-after asymptotic solutions. For instance, recognizing and separating the resonant degrees of freedom, identifying or defining a suitable reference system, selecting and properly ordering all the significant coupling and disorder parameters are key points which might benefit from a systematization and generalization effort, especially if accompanied by mechanical considerations and oriented to automatic solution algorithms.

According to these underlying motivations, the leading idea of the present paper is to systematize the multi-parameter approach to achieve a completely general purpose, that is, to perform eigensolution sensitivity analyses of multi-degree-of-freedom Hamiltonian systems in the parameter regions characterized by near-resonance conditions. No limitations on the discrete system dimension, nor on the number of nearly-resonant frequencies, are imposed a priori. The proposed algorithm requires minimal sufficient information consisting of the eigensolution (not necessarily complete) of a single nearly-resonant system, as could be derived for instance from experimental measurements. As a working hypothesis, the nearly-resonant system is postulated to arise from the unknown perturbation of a perfectly-resonant, though itself unknown, non-defective system. The paper is organized as follows. In Section 1 the general problem is posed, distinguishing two different and complementary tasks to be separately addressed. In Section 2 the eigensolution sensitivity of the perfect system, with respect to a generic multi-parameter perturbation, is analyzed (task I: *Direct problem*). The direct problem solution gives the approximated eigenpairs of each nearly-resonant system which could arise from the perturbation. The eigensolution sensitivity to disorder and coupling terms is separately discussed. In Section 3 the unknown perfect system to be perturbed is identified, starting from the knowledge of the experimental system (task II: *Inverse problem*). Questions regarding the existence and uniqueness of the inverse problem solution are discussed, consistent with the working hypothesis. Finally, in Section 4 the procedure efficacy is successfully tested on a prototypical structural system representing the sectional model of a suspended bridge by performing sensitivity analyses of the resonant eigensolution in the critical parameter regions.

1. Eigensolution of nearly-resonant systems

In structural mechanics, the linear dynamic model of a generic Hamiltonian system $\mathcal{S}$ with N-degrees-of-freedom (*dofs*) is governed by a pair of real-valued symmetric $N \times N$ matrices, the mass $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$. To the purposes of the present paper, the mass matrix can be assumed to be unitary ($\mathbf{M} = \mathbf{I}$), for the sake of simplicity. Thus the stiffness matrix depends on the complete set of M independent, dimensionless physical parameters, collected in the vector $\mathbf{p}$.