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Sensitivity analysis and optimization of vibration modes in continuum systems

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ABSTRACT

The sensitivity analysis of objective functions including the eigenmodes of continuum systems governed by scalar Helmholtz equations is carried out in continuum form. In addition, based on the sensitivity, the mode shapes are specified through numerical optimization. Using the continuum sensitivity and adjoint equation, the physical nature of them can be analyzed, which helps to explain the nature of the target optimization problem. Moreover, the continuum sensitivity and adjoint equation contribute to the quick numerical implementation of sensitivity analysis using software that can solve an arbitrary partial differential equation directly. A scalar Helmholtz equation in 1D or 2D domain is considered. The sensitivity analysis is performed for the general objective function formulated as a function of the eigenmode in continuum form. A minimization problem using the least squared error (i.e., difference) between the eigenvector and target mode shape is set as a sample objective function for both the first and second eigenmodes. The sensitivity and the adjoint equation are derived for this objective function. 1D and 2D numerical sensitivity analysis and optimization examples are studied to illustrate the validity of the derived sensitivity.

1. Introduction

The dynamic characteristics are one of the most important sets of properties in mechanical devices. To design devices with specified dynamic characteristics, various numerical optimization techniques were proposed in the context of a structural optimization field. In terms of the mechanical vibration, the most fundamental design specification is to avoid external disturbance vibrations resonating in the device. The fundamental method of achieving this is to maximize the lowest eigenfrequency. Starting from the early work in conventional shape optimization and grillage layout optimization [1,2], the homogenization or solid isotropic material with penalization (SIMP) based topology optimization [3,4] was applied to the problem [5–9]. Moreover, it was studied in various structural optimization methodologies as a benchmark problem [10–13]. Specifying an eigenfrequency for the target value [14] is also an effective way of avoiding resonance. Another approach for optimizing the response against an external dynamic load is the optimization of output displacement based on frequency response analysis [15–18] or time transient response analysis [19–21]. The maximization of the gap of the eigenfrequencies is also an interesting branch of the eigenfrequency optimization [10,22].

Eigenmodes are also an important factor in the optimization of vibration characteristics. For example, as proposed in [7], the eigenmode shape is used to track the desired vibration during the optimization entailing eigenmode switching

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based on the modal assurance criterion (MAC). Moreover, aside from the above work, which regards vibration as a phenomenon to be avoided, some research has proposed the optimization methodology of the vibration resonators that uses vibrations for the mechanical function [23–26]. In this type of resonator, the eigenmode that dominates the shape of deformation against the external periodic load is an important design factor in addition to the resonance frequency. Some research had difficulty with the optimization of the shape of the vibration mode [25,27,28]. Their approach was based on the discretization of the original continuum problem to a discrete problem using the finite element method (FEM). This discretization enabled the optimization of eigenmodes based on the eigenvector sensitivity analysis of matrices [29–31].

The concept of sensitivity analysis is not limited to discrete systems. As treated in some textbooks, the sensitivity and adjoint equations can be derived in the form of a continuum (e.g., [32–36]). This is the fundamental sensitivity analysis of continuum systems independent of discretization using numerical methods. By performing the sensitivity analysis in continuum form, an adjoint equation similar to the state equation can be obtained. Using this equation, the physical sense of the sensitivity and the adjoint equation can be analyzed as in [36]. This must be helpful for researchers and engineers studying the nature of vibration optimization. Moreover, in some commercial or open-source software, an arbitrary partial differential equation can be directly solved numerically by writing the PDE directly in the software [37,38]. If the adjoint equation is derived in continuum form, it can be directly solved using such software without self-produced code. Thus, it is necessary to broaden the design freedom of devices using external excitation vibrations. However, to the best of the authors' knowledge, sensitivity analysis in continuum form for the optimization problem of eigenmodes of continuum systems was studied only by [39]. In [39], the adjoint variable is calculated through modal analysis. Thus, the derivation of eigenmode sensitivity requires eigenfrequencies and eigenmodes that are of higher order than the target eigenfrequencies and eigenmodes. This must increase the computational cost of the optimization simply for solving the adjoint equation.

In this research, we derive the continuum sensitivity of the objective function including the eigenmodes of continuum systems governed by the scalar Helmholtz equations without using a modal method. Numerical optimization for the simple 1D and 2D vibration problems are also performed. That is, a scalar Helmholtz equation in the 1D or 2D domain is first considered. Then the sensitivity analysis is performed according to the procedure shown in [36]. A sample objective function is set as a minimization problem of the least square error of the first or second eigenvector and the target function. For this objective function, the sensitivity and the adjoint equation are derived in continuum form. The simple 1D string and 2D membrane vibration problems are set as numerical examples. The Helmholtz equation and the adjoint equation are solved numerically using the FEM for these problems. The sensitivity is calculated using the obtained state and adjoint variables. The results are compared with the sensitivity derived by the finite difference method to illustrate the validity of the derived sensitivity. Finally, some optimization problems are solved based on the derived sensitivity.

2. Formulation

2.1. Equations of state

In this research, the following simple scalar wave equation in the 1D or 2D domain Ω is considered as the equation of state

$$\frac{\partial^2 U(\mathbf{x},t)}{\partial t^2} = c(\mathbf{x})\nabla^2 U(\mathbf{x},t) \quad \text{in } \Omega \tag{1}$$

$$U(\mathbf{x},t) = 0 \quad \text{on } \Gamma_{\mathrm{D}} \tag{2}$$

where U is a scalar function with respect to time and space (representing the height of wave), c is the coefficient function defined in the whole domain and Γ_{D} defines the boundaries on which the Dirichlet condition is imposed with respect to u. The equations correspond to the free vibration of the strings or membranes, axial or torsion vibration of rods, etc. Assuming the time harmonic solution $U(\mathbf{x},t) = e^{-\mathrm{i}\omega t}u(\mathbf{x})$ where ω is the frequency and U is the amplitude, the above wave equation is converted into the following Helmholtz equation:

$$-\lambda u(\mathbf{x}) = c(\mathbf{x})\nabla^2 u(\mathbf{x}) \tag{3}$$

$$u(\mathbf{x}) = 0 \quad \text{on } \Gamma_{\mathrm{D}} \tag{4}$$

where

$$\lambda = \omega^2 \tag{5}$$

where λ is the eigenvalue. For the sensitivity analysis and numerical calculation using the FEM, the variational form of the above equation is also derived as follows:

$$a(u,v) - \lambda b(u,v) = 0 \tag{6}$$

where

$$a(u,v) = \int_{\Omega} c\nabla u \cdot \nabla v \, \mathrm{d}x \tag{7}$$