



## Technical paper

# Reliability and sensitivity analysis of a repairable system with imperfect coverage under service pressure condition

Kuo-Hsiung Wang<sup>a,\*</sup>, Tseng-Chang Yen<sup>b</sup>, Jen-Ju Jian<sup>b</sup>

<sup>a</sup> Department of Computer Science and Information Management, Providence University, Taichung 43301, Taiwan

<sup>b</sup> Department of Applied Mathematics, National Chung-Hsing University, Taichung 40227, Taiwan

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## ABSTRACT

This paper investigates reliability and sensitivity analysis of a repairable system with imperfect coverage under service pressure condition. Failure times and repair times of failed units are assumed to be exponentially distributed. As a unit fails, it may be immediately detected, located and replaced with a coverage probability  $c$  by a standby if one is available. When the repairmen are under the pressure of a long queue, the repairmen may increase the repair rate to reduce the queue length. We derive the explicit expressions for reliability function and mean time to system failure (*MTTF*). Various cases are analyzed to study the effects of different parameters on the system reliability and *MTTF*. We also accomplish sensitivity analysis and relative sensitivity analysis of the reliability characteristics with respect to system parameters.

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## 1. Introduction

Uncertainty is one of the important issues in management decisions. One of the most useful uncertainty measures is system reliability. The reliability of a system with standbys plays an important role in power plants, manufacturing systems, industrial systems and technical systems. Keeping a stable operating quality and a high level of reliability or availability is often a fundamental necessity. It maybe switches incompletely an existing spare module to a failed unit. When a failed unit is not detected, located and recovered, it needs time to be found and cleared. Therefore, we study the reliability of a system with multiple active units when covering a failed unit imperfectly. When the repairmen are under the pressure of a long queue, the repairmen may increase the repair rate to reduce the queue length. It may be impossible to switch in an existing spare module and then recover from a failure. Faults such as these are called to be *not covered*, and the probabilities of successful recovery on the failure of an active unit (or standby unit) is denoted by  $c$ . Quantity  $c$  which includes the probabilities of successful detection, location, and recovery from a failure is known as the *coverage factor* or *coverage probability* (see Trivedi [11]). A standby unit is called a 'warm standby' if its failure rate is nonzero and is less than the failure rate of an active unit. Active and warm

standby units can be considered to be repairable. We continue with the assumption that the coverage factor is the same for active and standby unit failures.

This paper differs from previous works in that: (i) the reliability problem with standby units has distinct characteristics which are different from the machine repair problem with standby units; (ii) it considers multiple imperfect coverage and reboot delay; and (iii) it considers the service pressure condition to prevent a long queue. The purpose of this article is to accomplish three objectives. The first objective is to develop the explicit expressions for reliability function,  $R_Y(t)$ , and mean time to system failure, *MTTF* using Laplace transform techniques. The second objective is to perform sensitive analysis and relative sensitivity analysis of  $R_Y(t)$  and *MTTF* along with specified values of the system parameters. The third objective is to provide the numerical results to illustrate the sensitivity and the relative sensitivity of  $R_Y(t)$  and *MTTF* with respect to system parameters.

Buzacott and Shanthikumar [2] reviewed queueing models that can be utilized to design manufacturing systems. The problem of machine failures and repairs in the context of queueing models has been investigated by several researchers. Govil and Fu [9] provided an excellent overview of the contributions of queueing models applied to manufacturing systems. In most papers, the queueing problems of the system discussed are more than the reliability problems of the system. Past work may be divided into two parts according to the system studied from the viewpoint of the queueing theory or from the viewpoint of the reliability. The major literature

\* Corresponding author. Fax: +886 4 26324045.

E-mail address: [khwang@pu.edu.tw](mailto:khwang@pu.edu.tw) (K.-H. Wang).

we review is the viewpoint of the reliability system. Cao and Cheng [3] first introduced reliability concept into a queueing system with a repairable service station where the lifetime of the service station is exponentially distributed and its repair time has a general distribution. Wang and Sivazlian [16] presented the reliability characteristics of a system consisting of  $M$  operating machines,  $S$  warm standbys and  $R$  repairmen. They have established relations between system reliability and the number of spares, the number of repairmen, the failure rate and the repair rate. Meng [10] compared the mean time to system failure for four redundant series. He obtained a general ordering relationship between the  $MTTF$  of these four systems. Wang and Kuo [14] investigated the reliability and availability characteristics of four different series system configurations with mixed standby components. They have provided a systematic methodology to develop the  $MTTF$  and the steady-state availability of four configurations with mixed standby units. Galikowsky et al. [4] and Wang and Pearn [15] investigated the cost benefit analysis of series system with cold standby components and warm standby components, respectively. They developed the explicit expressions for the  $MTTF$  and the steady-state availability. Ke and Wang [6] extended Wang and Sivazlian's [16] model by considering the balking and reneging in a repairable system. They provided the explicit expressions for the reliability characteristics of a repairable system with warm standby units plus balking and reneging. The concept of coverage factor and its effect on the reliability and availability model of a repairable system has been introduced by several authors such as Arnold [1], Trivedi [11], and Wang and Chiu [13]. The idea of imperfect coverage we discussed in this paper has been proposed by Trivedi [11]. Moreover, Wang and Chiu [13] analyzed the cost benefit analysis of availability systems with warm standby units and imperfect coverage. The concept of reboot delay and its effect on the reliability and availability model of a repairable system has been introduced by Trivedi [11]. Recently, Wang and Chen [12] investigated the reliability and availability analysis of a repairable system with standby switching failures. When the repairmen are under the pressure of a long queue, they may increase the repair rate to reduce the queue length. The concept of the service pressure coefficient was first introduced by Hiller and Lieberman [5]. Recently, Ke et al. [8] studied the reliability and sensitivity analysis of a system with multiple unreliable servers and standby switching failures. Ke et al. [7] first studied the reliability analysis of a system with standbys subjected to switching failure and presented a contour of the  $MTTF$  which is useful for the decision makers.

**2. Description of the system and notations**

The repairable system includes  $M$  active units,  $S$  warm standby units and  $R$  reliable repairmen. The queue discipline of this repairable system is assumed to be FCFS (first-come first-serve). Each of the active units has an exponential time-to-failure distribution with parameter  $\lambda$ . Each of the warm standby units also has an exponential time-to-failure distribution with parameter  $\alpha$  ( $0 < \alpha < \lambda$ ). When a unit fails, it may be immediately detected, located and replaced with a coverage probability  $c$  by a standby if one is available. It is assumed that the replacing time is instantaneous. We define the unsafe failure state of the system as any one of the breakdowns is not covered. Active unit with failure in the unsafe failure state is refreshed by a reboot. Reboot delay takes place at rate  $\beta$  for an active unit (or standby unit) which is exponentially distributed. The repair rate changes by the large number of failed units for waiting to repair. Therefore, we consider the service pressure coefficient  $a$  to improve the repair rate. Service pressure coefficient is a positive constant and indicates degree to which repair rate is affected by the number of failed units in the system. System failure is defined to be less than  $K$  active units, where  $K = 1, 2, \dots, M$ .

Therefore, if  $n$  denotes the number of failed units in the system, the system is failed if and only if  $n \geq L = M + S - K + 1$ . In this paper, we consider two cases for system failure: Case 1, the system fails when all  $M + S$  units fail (i.e.,  $K = 1$ ); and Case 2, the system fails when the standby units are emptied (i.e.,  $K = M$ ).

**Notations**

$M$	number of active units
$S$	number of warm standby units
$R$	number of repairmen
$n$	number of failed units in the system
$s$	Laplace transform variable
$\lambda$	failure rate of an active unit
$\alpha$	failure rate of a warm standby unit
$\mu$	repair rate of a failed unit
$c$	coverage probability
$\beta$	reboot delay rate
$a$	service pressure coefficient
$P_n^*(s)$	Laplace transform of $P_n(t)$
$P_{ufn}^*(s)$	Laplace transform of $P_{ufn}(t)$
$R_Y(t)$	reliability function of the system
$MTTF$	mean time to system failure

**3. Reliability characteristics**

At time  $t = 0$ , the system has just started operation with no failed units when the repairman is working. The reliability function under exponential failure time, exponential reboot time and exponential repair time can be developed through the birth and death process.

Let  $n$  denote the number of failed units in the system. The mean failure rate  $\lambda_n$  and the mean service rate  $\mu_n$  for this model are given by

$$\lambda_n = \begin{cases} M\lambda + (S - n)\alpha, & 0 \leq n \leq S - 1 \\ (M + S - n)\lambda, & S \leq n \leq M + S. \end{cases} \tag{1}$$

$$\mu_n = \begin{cases} n\mu, & 1 \leq n \leq R - 1 \\ R\mu \left[ \frac{n(R + 1)}{R(n + 1)} \right]^a, & R \leq n \leq M + S. \end{cases} \tag{2}$$

*3.1. Differential-difference equations*

Let us define  $P_n(t)$  is the probability of exactly  $n$  failed units in the system at time  $t$  in the safe state and  $P_{ufn}(t)$  is the probability of exactly  $n$  failed units in the system at time  $t$  in the unsafe failure state.

From the state-transition-rate diagram shown in Fig. 2, we set up the differential difference equations as follows:

$$\frac{dP_0(t)}{dt} = -\lambda_0 P_0(t) + \mu_1 P_1(t) \tag{3}$$

$$\frac{dP_n(t)}{dt} = -(\lambda_n + \mu_n)P_n(t) + \lambda_{n-1}cP_{n-1}(t) + \beta P_{ufn}(t) + \mu_{n+1}P_{n+1}(t), \tag{4}$$

$$1 \leq n \leq L - 2$$

$$\frac{dP_{ufn}(t)}{dt} = -\beta P_{ufn}(t) + \lambda_{n-1}(1 - c)P_{n-1}(t), \tag{5}$$

$$1 \leq n \leq L - 1$$

$$\frac{dP_{L-1}(t)}{dt} = -(\lambda_{L-1} + \mu_{L-1})P_{L-1}(t) + \lambda_{L-2}cP_{L-2}(t) + \beta P_{ufL-1}(t), \tag{6}$$

$$\frac{dP_L(t)}{dt} = \lambda_{L-1}P_{L-1}(t). \tag{7}$$

Eqs. (3)–(7) can be written in matrix form as:

$$\dot{P}(t) = Q \cdot P(t), \tag{8}$$

where  $P(t)$  denotes the column vector  $[P_0(t), P_{uf1}(t), P_1(t), \dots, P_{L-1}(t), P_{ufL-1}(t), P_L(t)]^T$ .

Note that the symbol  $T$  denotes the transpose.  $\dot{P}(t)$  indicates the derivative of  $P(t)$  with respect to  $t$  and  $Q$  is the characteristic matrix

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