An application of the Kriging method in global sensitivity analysis with parameter uncertainty

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Abstract

For structural systems with both epistemic and aleatory uncertainties, the effect of epistemic uncertainty on failure probability is measured by the variance based sensitivity analysis, which generally needs a “triple-loop” crude sampling procedure to solve and is time consuming. Thus, the Kriging method is employed to avoid the complex sampling procedure and improve the computational efficiency. By utilizing the Kriging predictor model, the conditional expectation of failure probability on the given epistemic uncertainty can be calculated efficiently. Compared with the Sobol’s method, the proposed one can ensure reasonable accuracy of results but with lower computational cost. Three examples are employed to demonstrate the reasonability and efficiency of the proposed method.

1. Introduction

Sensitivity analysis (SA) is widely used in engineering design, which can be classified into two groups: local SA and global SA [1]. Local SA techniques are usually investigated how small variations of parameters around a nominal point change the value of the output. The main disadvantages of them are that they depend on the choice of the nominal point in the parameters space. Global SA takes into account all the variation range of the parameters, and apportions the output uncertainty to the uncertainty of the input parameters, covering their entire range space [2]. At present, a number of measures have been suggested, such as, Helton and Saltelli [3,4] proposed the nonparametric techniques (input–output correlation), Sobol, Iman and Saltelli [4–6] proposed a series of variance based importance measures, Chun, Liu and Borgonovo [7,8] proposed moment independent sensitivity indicators. But those indicators are all proposed for structural system with epistemic input uncertainty. Hofer and Krzykaca-Hansmann [9,10] investigated another situation that the input uncertainty of a model is only aleatory uncertainty described by the probability distribution and the distributional parameters of inputs are not known precisely which are subject to epistemic uncertainty. In their works, they proposed the variance-based sensitivity measures in the presence of epistemic and aleatory uncertainties which can be used to identify the most influential distribution parameters. Based on this idea, we proposed the variance-based sensitivity measures of failure probability in the presence of epistemic and aleatory uncertainties, which can be used to identify the most influential distribution parameters on the safety of a system.

The variance based measures generally require a large number of function evaluations to achieve reasonable convergence and can become impractical for most engineering problems. Recently, several works estimate the variance based sensitivity measure with a group of given samples [11,12], but they still face the problem of “curse of dimensionality”. Thus this paper employs the Kriging method to overcome this problem, which has been widely used for deterministic optimization problems.
The Kriging method can be represented as an improved linear regression technique [15]. It consists of a parametric linear regression model and a nonparametric stochastic process, which can approximate the failure probability of a test point by the weighted average of the failure probability of training points surrounding the test point. The mapping relationship of epistemic parameters and failure probability can be obtained directly by the Kriging method, and then the conditional expectation of failure probability can be calculated conveniently which avoids the complex sampling procedure. The computational efficiency of the Kriging method can be validated by several numerical and engineering examples.

This paper is organized as follows: Section 2 analyzes the propagation of the uncertainty of the structural system and distinguishes the epistemic and aleatory uncertainties. Then the “black box” model of epistemic parameters and failure probability is given. Section 3 first employs the standard Sobol’s method to solve the variance based sensitivity measure, and then a novel method based on the Kriging model is proposed. Three examples, including a numerical example, a roof truss structure and an automobile front axle structure, are employed to validate the reasonability and efficiency of the proposed method in Section 4. Finally, conclusions are drawn in Section 5.

2. Global sensitivity measures

2.1. The propagation of epistemic and aleatory uncertainties

Considering the structural system with aleatory uncertainty, the performance function can be given as:

$$Y = g(X)$$

(1)

where $Y$ is the output response, $X = (X_1, X_2, ..., X_n)$ are $n$ independent inputs, which are variables subjected to aleatory uncertainty such as material, loads, geometry, etc. The aleatory uncertainty is caused by some accidental uncontrollable factors, and it cannot be decreased.

Considering the structural system with both epistemic and aleatory uncertainties, the performance function can be rewritten as:

$$Y = g(X, \theta)$$

(2)

where $\theta = (\theta_1, \theta_2, ..., \theta_p)$ are $p$ independent distribution parameters, which can be represented by random model in this paper, namely the epistemic uncertainty is described by probability density function $f_\theta(\theta)$. With the accumulation of the sample data and the improvement of knowledge, the epistemic uncertainty of distribution parameters can be decreased. When given a certain parameter vector $\theta^*$, the aleatory uncertainty of input $X$ can be described by the conditional probability density function $f_X(X|\theta^*)$. When the sufficient knowledge of parameter is unavailable, an approximate distribution assumption can be made such that the uncertainties of statistic characteristics are completely determined by their two central moments (and this is the case for almost all standard parametric distributions) [10]. Furthermore, an individual may alternatively apply the maximum entropy principle to arrive at a distribution having the two approximated central moments but otherwise having maximum epistemic uncertainty associated with it. Especially, if no further information about this distribution is available, then it needs to ask experts for their subjective distributions for the unknown parameters. These distributions may be uniform (no most likely value), triangular, normal, lognormal, etc. In our paper, we primarily investigate the typical normal distribution with the given mean $\mu$ and the given standard deviation $\sigma$. This assumption is made in our examples to represent the epistemic uncertainty.

It is noticed that the uncertainty of parameters can lead to the uncertain of the statistic characteristics of output, such as expectation, variance, failure probability, etc. In order to measure the effect of the epistemic uncertainty on the safety of structural system, the mathematical mapping between failure probability and distribution parameters can be given as:

$$P_f = \psi(\theta)$$

(3)

It can be seen from Fig. 1 that the epistemic uncertainty of distribution parameters can affect the aleatory uncertainty of inputs by conditional probability density function $f_X(X|\theta^*)$, then the aleatory uncertainty can lead to the uncertain of output by performance function, and finally affect the uncertainty of failure probability. However, the mapping relationship of distribution parameters and failure probability is complex, it cannot be given analytically [16], thus, the “black box” model $\psi$ is employed to describe this relationship and there is the one to one correlation between distribution parameters and failure probability.

![Fig. 1. Depict for the uncertainty propagation of structural system.](image-url)
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