

Direct differentiation method for response sensitivity analysis of a bounding surface plasticity soil model

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ARTICLE INFO

Article history:

Received 17 February 2012

Received in revised form

26 January 2013

Accepted 31 January 2013

Available online 15 March 2013

ABSTRACT

Finite element (FE) response sensitivity analysis is an important component in gradient-based structural optimization, reliability analysis, system identification, and FE model updating. In this paper, the FE response sensitivity analysis methodology based on the direct differentiation method (DDM) is applied to a bounding surface plasticity material model that has been widely used to simulate nonlinear soil behavior under static and dynamic loading conditions. The DDM-based algorithm is derived and implemented in the general-purpose nonlinear finite element analysis program OpenSees. The algorithm is validated through simulation of the nonlinear cyclic response of a soil element and a liquefiable soil site at Port Island, Japan, under earthquake loading. The response sensitivity results are compared and validated with those obtained from Forward Finite Difference (FFD) analysis. Furthermore, the results are used to determine the relative importance of various soil constitutive parameters to the dynamic response of the system. The DDM-based algorithm is demonstrated to be accurate and efficient in computing the FE response sensitivities, and has great potential in the sensitivity analysis of nonlinear dynamic soil-structure systems.

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1. Introduction

Finite element (FE) response sensitivity analysis is an essential ingredient of gradient-based optimization methods and is required in structural optimization, system identification, reliability, and FE model updating [1–4]. Furthermore, the sensitivity analysis results may be used to propagate the material and loading uncertainty to the structural responses of interest. In addition, FE response sensitivities provide invaluable insight into the effects of system parameters on, and their relative importance to, the system response [5]. Several methods are available for response sensitivity analysis, including the Finite Difference Method (FDM), the Adjoint Method (AM), the Perturbation Method (PM), and the Direct Differentiation Method (DDM) [6–11]. The FDM is the simplest method for response sensitivity computation, but is computationally expensive and can be negatively affected by numerical noise. The AM is efficient for linear and non-linear elastic systems, but is not a competitive method for path-dependent (i.e., inelastic) problems. The PM is computationally efficient but generally not very accurate. The DDM, on the other hand, is a general, accurate and efficient method that is applicable to any material constitutive model. The DDM-based

response sensitivity analysis methodology shows great promise in the analysis of large and complex structural or geotechnical systems.

However the DDM method requires analytical derivations and their computer implementation to differentiate the system responses with respect to sensitivity parameters. Over the past decade, the DDM-based sensitivity analysis method has been actively developed and implemented in an open source FE analysis framework known as OpenSees [12]. The DDM has been developed for various constitutive models including uniaxial materials, three-dimensional J_2 plasticity models and pressure-independent multi-yield surface J_2 plasticity models [13]. These models can be used to simulate truss and beam components in structures, and nonlinear clay behaviors. Detailed descriptions of the DDM-based sensitivity analysis methodology implemented in OpenSees can be found in the literature [14–17].

Yet, the method has not been formulated for sandy soils, which usually exhibit different behavior from clayey soils, such as pressure-dependent cyclic behaviors, shear-induced volumetric dilation and contraction, as well as liquefaction under low effective confinement. The objective of this paper is to extend the DDM-based sensitivity analysis to a class of bounding surface models for sandy soils. The bounding surface model has been widely used and proven to be an effective and robust model to simulate the behaviors of sandy materials under cyclic and seismic loading conditions [18–21]. The DDM-based sensitivity algorithm is particularly

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efficient for strongly nonlinear, large-scale problems with a large number of sensitivity parameters. Geotechnical problems modeled using the bounding surface model are such examples. Thus developing a DDM-based sensitivity algorithm for the bounding surface model will allow us to solve a large number of challenging geotechnical problems, such as the earthquake-induced liquefaction phenomenon in sandy soils. When combined with the existing sensitivity analysis framework for clayey soils and soil-structure systems, the DDM-based sensitivity analysis may be readily applied to real soil-foundation-structure interaction systems [17].

This paper provides a summary of the bounding surface model and detailed DDM formulation, followed by examples to validate the DDM-based FE response sensitivity algorithm. The algorithm is applied to study the sensitivity of liquefied ground responses observed at Port Island in Japan under a real earthquake scenario. The results are further used to identify the relative importance of the soil parameters to the ground surface response.

2. Numerical implementation of a bounding surface model

The bounding surface model presented herein was developed for simulating the pressure-dependent behaviors of sandy soils under complex loading conditions [18,19]. Compared with the classical plastic theory using yield surfaces, flow rules and hardening laws to characterize the plastic behavior of a material, this model generalizes the yield-surface-based plasticity theory by defining a bounding surface or a failure surface. The plastic deformation within the bounding surface is determined by a varying plastic modulus, which is defined as a continuous function of the distance from the current stress to a properly defined ‘image’ stress on the bounding surface. The model was further improved to incorporate the basic premises of critical-state soil mechanisms to allow for the realistic modeling of the shear-induced volumetric changes (i.e., contraction or dilation) in either a loose or a dense state, and the phase transition from one state to another [20,21], which is the basis for modeling the liquefaction behavior of sandy soils. In practice, this model has been implemented in some commercial softwares, and verified using extensive experimental data and real earthquake records [22].

2.1. Constitutive formulation

The bounding surface model employs a stress ratio invariant, defined as $R = (\frac{1}{2}\mathbf{r}:\mathbf{r})^{1/2}$, where \mathbf{r} is the stress ratio of the deviatoric stress \mathbf{s} over pressure p , i.e., $\mathbf{r} = \frac{\mathbf{s}}{p}$, and the notation “:” is the double contraction between two second-order tensors, i.e., $\mathbf{A}:\mathbf{B} = A_{ij}B_{ij}$. Accordingly, an ultimate failure surface, or a failure-bounding surface, is defined as

$$\hat{f} = R - R_f = 0 \quad (1)$$

where the parameter R_f is the stress ratio invariant at the failure surface, which is related to the corresponding classical critical state triaxial parameter M by $R_f = M/\sqrt{3}$. Stress is not allowed to trespass the failure-bounding surface $\hat{f} = 0$. Similarly, the maximum prestress memory bounding surface is defined as:

$$\bar{f} = R - R_m = 0 \quad (2)$$

where R_m is a history parameter providing the maximum prestress level. The two bounding surfaces $\hat{f} = 0$ and $\bar{f} = 0$ are combined to compute the plastic modulus.

Inside the failure-bounding surface, the hypoelastic response, i.e., the elastic strain rate $\dot{\boldsymbol{\epsilon}}^e$, is defined as the summation of

deviatoric strain $\dot{\boldsymbol{\epsilon}}^e$ and volumetric strain $tr\dot{\boldsymbol{\epsilon}}^e$:

$$\dot{\boldsymbol{\epsilon}}^e = \dot{\boldsymbol{\epsilon}}^e + \frac{1}{3}(tr\dot{\boldsymbol{\epsilon}}^e)\mathbf{I} = \frac{1}{2G}\dot{\mathbf{s}} + \frac{1}{3K}\dot{p}\mathbf{I} = \frac{1}{2G}p\dot{\mathbf{r}} + \left(\frac{1}{2G}\mathbf{r} + \frac{1}{3K}\mathbf{I}\right)\dot{p} \quad (3)$$

where G and K are the pressure-dependent elastic shear and bulk moduli, respectively. Similarly, the hypoelastic response, i.e., the plastic strain rate $\dot{\boldsymbol{\epsilon}}^p$, can be written as

$$\dot{\boldsymbol{\epsilon}}^p = \left(\frac{1}{H_r}\mathbf{n}_D + \frac{1}{3K_r}\mathbf{I}\right)(p\dot{\mathbf{r}}:\mathbf{n}_N) + \left(\frac{1}{H_p}\mathbf{r} + \frac{1}{3K_p}\mathbf{I}\right)h(p-p_m)\langle\dot{p}\rangle \quad (4)$$

where H_r and K_r are, respectively, the plastic shear and bulk moduli associated with the deviatoric stress ratio \mathbf{r} ; parameters H_p and K_p are, respectively, the plastic shear and bulk moduli associated with the pressure rate \dot{p} . The vectors \mathbf{n}_D and \mathbf{n}_N are unit vectors in stress space along the deviatoric part of $\dot{\boldsymbol{\epsilon}}^p$ and the associated deviatoric loading direction, respectively. In this paper both \mathbf{n}_D and \mathbf{n}_N are taken to be the same as the unit vector normal to the maximum prestress memory bounding surface $\bar{f} = 0$ (i.e., vector \mathbf{n} in Fig. 1). The p_m is the maximum value of mean pressure p experienced in past loading. The Heaviside step function $h(p-p_m)$ and the Macaulay brackets $\langle\cdot\rangle$ around \dot{p} indicate that the plastic mechanism due to \dot{p} operates only when $p = p_m$ and $\dot{p} > 0$. As shown in Fig. 1, the previous unloading stress point (i.e., $\boldsymbol{\alpha}$ in Fig. 1), the current deviatoric stress ratio \mathbf{r} and a properly defined ‘image’ stress $\bar{\mathbf{r}}$ on the maximum prestress memory bounding surface $\bar{f}(\boldsymbol{\sigma}) = 0$ are combined to determine variable plastic moduli H_r and K_r , which are continuous functions of the distance ρ from $\boldsymbol{\alpha}$ to \mathbf{r} ($\rho = \|\mathbf{r} - \boldsymbol{\alpha}\|_2$) and the distance $\bar{\rho}$ from $\boldsymbol{\alpha}$ to $\bar{\mathbf{r}}$ ($\bar{\rho} = \|\bar{\mathbf{r}} - \boldsymbol{\alpha}\|_2$) [18]. It is worth mentioning that for practical applications, the shear-induced plastic strains usually dominate. Therefore, the second term in Eq. (4), i.e., $\left(\frac{1}{H_p}\mathbf{r} + \frac{1}{3K_p}\mathbf{I}\right)h(p-p_m)\langle\dot{p}\rangle$, is neglected in this paper for simplicity. The plastic strain rate $\dot{\boldsymbol{\epsilon}}^p$ can be simplified as:

$$\dot{\boldsymbol{\epsilon}}^p = \left(\frac{1}{H_r}\mathbf{n} + \frac{1}{3K_r}\mathbf{I}\right)(p\dot{\mathbf{r}}:\mathbf{n}) \quad (5)$$

2.2. Numerical implementation

The numerical implementation of the constitutive model employs an explicit algorithm for computing the plastic moduli H_r and K_r . In this section, the discretized version of the constitutive model is presented in detail. The variables with subscript n denote the ones at the last time step at discrete time t_n .

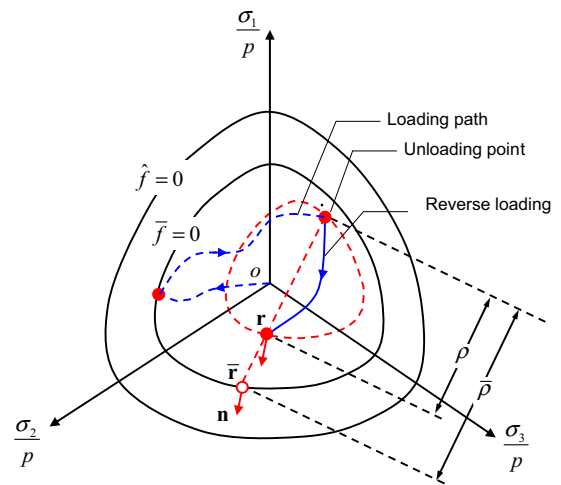


Fig. 1. The bounding surface model in deviatoric stress ratio space.

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