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Refractive index sensitivity analysis of long period fiber grating by new transfer matrix method

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ABSTRACT

The refractive index sensitivity of long period fiber grating is analyzed by new transfer matrix method. Compared to traditional transfer matrix method, The new transfer matrix method can be used to analyze the modes coupling between the core mode and multiple cladding modes through only one time calculation. Compared with the previous method used, such as solving the coupled mode equation by the fourth order adaptive step size Runge-Kutta algorithm, the new transfer matrix method has a faster calculation speed. Theoretical results are excellent agreement with the method of solving the coupled mode equation.

1. Introduction

Long period fiber gratings (LPFGs) have been found many applications in optical telecommunications and optical sensor such as mode converters [1], rejection filters [2], gain-flattening filters for erbium-doped fiber amplifiers [3], optical fiber sensors for strain [4], temperature [5], and refractive index measurements [6] because of their capability to couple power between core and cladding modes at resonant wavelengths [7–10].

The refractive index sensitivity of LPFG can be obtained by analyzing spectral variation under a certain ambient index changes. The common methods used to analyze the spectral characteristics of LPFG and fiber Bragg gratings include transfer matrix method (TMM) and solving the coupled mode equations (SCME) [11,12]. The traditional TMM is able to analyze the uniform and non-uniform LPFG and fiber Bragg gratings when only two modes are considered [11,12]. The SCME method can obtain the spectrum of the uniform and non-uniform structure LPFG when multiple modes are considered. In this paper, a new TMM about LPFG with multiple cladding modes coupled is proposed and applied to analyze the refractive index sensitivity of LPFG. Compared to traditional TMM, the advantage of the new TMM is that multiple cladding modes can be considered through only one time calculation for analyzing the transmission characteristic of LPFG. This new TMM can be used to analyze the modes coupled both about the uniform and the non-uniform between the core mode and the multiple cladding modes. And the new TMM is simple to implement, almost always sufficiently accurate, and generally faster than that of SCME.

2. New transfer matrix

The coupled-mode equations of LPFG are given as [11]

$$\begin{cases} \dfrac{dA^{co}}{dz} = jk^{co-co}_{01-01}A^{co} + j\sum\limits_{\nu} \dfrac{m}{2}k^{cl-co}_{1\nu-01}A^{cl}_{\nu} e^{-j2\delta^{cl-co}_{1\nu-01}z} \\ \sum\limits_{\nu} \left[\dfrac{dA^{cl}_{\nu}}{dz} = j\dfrac{m}{2}k^{cl-co}_{1\nu-01}A^{co}e^{j2\delta^{cl-co}_{1\nu-01}z}\right] \end{cases} \tag{1}$$

where A^{co} is the amplitude for the core mode, A^{cl} is the amplitude for the cladding mode $HE_{1\nu}$, k^{co-co}_{01-01} is the coupling constant for core-mode–core-mode, $k^{cl-co}_{1\nu-01}$ is the coupling constant for core-mode–cladding-mode.

$$\delta^{cl-co}_{1\nu-01} = \frac{1}{2}\left(\beta^{co}_{01} - \beta^{cl}_{1\nu} - \frac{2\pi}{\Lambda}\right) \tag{2}$$

where β^{co}_{01} and $\beta^{cl}_{1\nu}$ are the propagation constant of the core mode and cladding mode, and Λ is the period of grating.

If defined S_{ν} as:

$$S_{\nu} = A^{cl}_{\nu}e^{-j2\delta^{cl-co}_{1\nu-01}z} \tag{3}$$

follow equations can be obtained as:

$$\begin{cases} \dfrac{dA^{co}}{dz} = jk^{co-co}_{01-01}A^{co} + \sum\limits_{\nu} j\dfrac{m}{2}k^{cl-co}_{1\nu-01}S_{\nu} \\ \sum\limits_{\nu} \left[\dfrac{dS_{\nu}}{dz} = j\dfrac{m}{2}k^{cl-co}_{1\nu-01}A^{co} - j2\delta^{cl-co}_{1\nu-01}S_{\nu}\right] \end{cases} \tag{4}$$

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then the matrix form of formula (4) can be expressed as:

$$\begin{bmatrix} \dfrac{dA^{co}}{dz} \\ \dfrac{dS_1}{dz} \\ \vdots \\ \dfrac{dS_\nu}{dz} \end{bmatrix} = F \begin{bmatrix} A^{co} \\ S_1 \\ \vdots \\ S_\nu \end{bmatrix} \tag{5}$$

If defined a vector A as:

$$A = \begin{bmatrix} A^{co} \\ S_1 \\ \vdots \\ S_\nu \end{bmatrix} \tag{6}$$

formula (4) can be expressed as:

$$\frac{dA}{dz} = FA \tag{7}$$

where

$$F = j \begin{bmatrix} k^{co-co}_{0,1-01} & \frac{m}{2}k^{cl-co}_{1,1-01} & \frac{m}{2}k^{cl-co}_{1,2-01} & \cdots & \frac{m}{2}k^{cl-co}_{1,\nu-1-01} & \frac{m}{2}k^{cl-co}_{1,\nu-01} \\ \frac{m}{2}k^{cl-co}_{1,1-01} & -2\delta^{cl-co}_{1,1-01} & 0 & \cdots & 0 & 0 \\ \frac{m}{2}k^{cl-co}_{1,2-01} & 0 & -2\delta^{cl-co}_{1,2-01} & 0 & \cdots & 0 \\ \vdots & \vdots & 0 & \ddots & 0 & \vdots \\ \frac{m}{2}k^{cl-co}_{1,\nu-1-01} & 0 & \vdots & 0 & -2\delta^{cl-co}_{1,\nu-1-01} & 0 \\ \frac{m}{2}k^{cl-co}_{1,\nu-01} & 0 & 0 & \cdots & 0 & -2\delta^{cl-co}_{1,\nu-01} \end{bmatrix} \tag{8}$$

For the uniform grating, F is a constant, formula (7) is a linear constant coefficient difference equation. The form of the solution of the formula (7) can be expressed as:

$$A(z) = A_0 e^{Fz} \tag{9}$$

where A_0 is the initial value of the vector at $z = 0$ position, and can be expressed as:

$$A_0 = [1, 0, \cdots, 0]' \tag{10}$$

so, the transfer matrix of the uniform LPFG can be expressed as:

$$T = e^{FL} \tag{11}$$

where L is the grating length.

So the transmission rate of core mode can be expressed as:

$$\rho = \frac{A^{co}(L)}{A^{co}(0)} = A^{co}(L) \tag{12}$$

For the non-uniform LPFG the grating can be divided into hundreds segments, and every segment can be considered as uniform. The transfer matrix of the ith segment can be expressed as:

$$T_i = e^{FL_i} \tag{13}$$

where L_i is the length of ith segment. The total transfer matrix of the grating can be expressed as:

$$T = T_0 \quad T_1 \cdots T_N \tag{14}$$

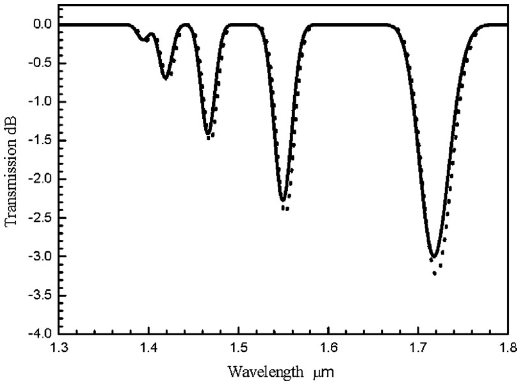

Fig. 1. Theoretically calculated transmission spectrum by new TMM (solid line) and SCME (dotted line) through a Blackman apodization grating.

3. Refractive index sensitivity analysis

In this section we analyze the refractive index sensitivity of LPFG by the new TMM as described above. The fiber considered in this paper is of a step-index profile and a three layers structure. The parameters of the fiber are given as: the core radius $r_1 = 2.625\ \mu\text{m}$, the cladding radius $r_2 = 62.5\ \mu\text{m}$, the core index $n_1 = 1.458$, the cladding index $n_2 = 1.45$ and the air index $n_3 = 1.0$.

Firstly, in order to prove the correctness of the new TMM, a Blackman apodization LPFG is analyzed. The transmission spectrum of this grating is shown in Fig. 1 (solid line). The grating parameters are given as: grating period $\Lambda = 470\ \mu\text{m}$, grating length $L = 25$ mm, the peak induced-index change is 0.0001. The five main dips shown in the spectrum are the result from the coupling between the core mode and the $\nu = 1, 3, 5, 7, 9$ cladding modes. For comparison, the transmission spectrum of this grating is calculated again by the SCME, which is illustrated in Fig. 1 (dotted line). From Fig. 1 it can be obtained that the new TMM is exactly enough to analyze the transmission spectrum of LPFG.

Compared with the SCME method, the advantage of the new TMM is that it can analyze the transmission characteristic of non-uniform LPFG with a faster speed. In order to compare the calculating speed of these two methods, the computing time is illustrated in Fig. 2 when different cladding modes have been considered. It can be found that the calculating speed by the new TMM is faster than that by SCME.

When the ambient refractive index varies, the cladding modes of the fiber will be changed. Fig. 3 shows the transmission spectral of LPFG when refractive index varied. The single resonance that is visible with in the wavelength range plotted is associated with coupling to the $\nu = 9$ cladding mode. This figure shows that the coupled wavelength will shift to the shorter wavelength when the refractive index raised. The more details on the wavelength shift about more cladding mode are illustrated in Fig. 4. From these figures