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# General continuous time models for production planning and scheduling of batch processing plants: mixed integer linear program formulations and computational issues

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**Abstract**

In this work, a continuous time model for optimal planning and scheduling of the production in batch processing plants is developed. The considered plants are general in the sense the products can run through different processing stages and follow different manufacturing routes. Processing stages are represented as operations. They can be viewed as modules of operators accomplishing same tasks and sharing possibly different operational characteristics. The resulting continuous time mixed integer nonlinear program (MINLP) is capable to handle complicating situations such as batch splitting, resource allocation and equipment maintenance. By using known linearization techniques, the MINLP is reformulated as a mixed integer linear program. It is further refined by using a modified version of the reformulation linearization technique and some other equivalent reformulations. The model is also implemented on a real life case: paint production. The computation of optimal production plan and schedule takes only a few minutes for this real case. © 2001 Elsevier Science Ltd. All rights reserved.

*Keywords:* Batch process; Planning; Scheduling; Continuous time model; Mixed integer linear program

## 1. Introduction

In commercial productions, the economical aspect is frequently the most important factor in choosing a processing technology, assuming that the technology is also safe and clean. In the middle two quarters of the twentieth century, continuous processing gained considerable popularity, since it is possible to take advantage of economies of scales. However, continuous processes are lacking processing flexibility; large and expensive plants have to be constructed to produce chemicals with tight specifications in large amounts. In addition, consumers have become more sophisticated and demand for chemical products with variable specifications has increased within the last quarter of the century. This fact has forced the focus again on highly flexible batch processing.

In many branches of chemical industry, such as traditional paint, food, pharmaceuticals, specialty chemicals, batch production schemes are always preferred to continuous processes because of their ability to respond quickly to frequently changing market conditions and their ease of control. Furthermore, while introducing a brand new product into the market, if the demand for that product is uncertain and not well established, which is true for such branches, a batch process requiring rather low investment is usually preferred.

Therefore, the production of a product with a single (most demanded) specification in large scale has become less desirable. Instead, producing products with varying specifications in small amounts has gained popularity because of the chance for a higher market share it offers. Thus, the demand for the products produced in high volumes with a single specification decreased, which in turn forced producers to produce a variety of recipes in smaller volumes. As a consequence, some of the continuous processes, which used to be popular in

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the second and third quarter of the century, are nowadays being replaced by batch production schemes.

On the other hand, developments in flexible production systems make the production of a wide-variety of specifications of a product (multi-product plants), the simultaneous production of many different products (multi-purpose plants), and the simultaneous production of a variety of specifications of many different products (multi-purpose multi-product plants) in a facility possible. These advantages flexible production systems offer, provide another important reason for the use of batch processing. As a result, batch production schemes still play a very important role in chemical industries, and they seem to continue to be a feasible processing mode in the foreseeable future.

To benefit fully from what batch processing schemes offer, in order to maximize the profit of the company, the management should efficiently coordinate the equipment (machines) and human resources, namely determine an optimal product mix, prepare efficient production plans and operation schedules. In short, planning and scheduling of a batch processing plant is a crucial problem. Unfortunately, optimal scheduling and planning of batch production plants is a very hard objective to achieve. In chemical industry, this difficulty arises mainly because of the large variety of processing equipment with varying operational and cost characteristics.

As a consequence, a general methodology to cope with this difficult but important problem, namely the determination of production plans and operation schedules in order to maximize profit, is one of the objectives of this work.

In the related literature, there is a remarkable number of results on the scheduling of batch process chemical plants. They may be divided into two major categories. The first group examines the effect of scheduling on plant design (Vaselenak, Grossmann, & Westerberg, 1987; Cerda, Vicente, Guiterrez, Esplugas, & Mata, 1989; Coulman, 1989; Birewar & Grossmann, 1990a; Pinto & Rao, 1992; Povua & Macchietto, 1992; Sahinidis & Grossmann, 1992; Voudouris & Grossmann, 1996), while the second group is interested in the scheduling of operations of existing plants (Ku & Karimi, 1988; Birewar & Grossmann, 1989, 1990b; Huisman, Polderman, & Weeda, 1990; Ku & Karimi, 1990, 1991a; Sahinidis & Grossmann, 1991; Cao & Bedworth, 1992; Vickson & Alfredsson, 1992; Kondili, Pantelides, & Sargent, 1993; Pekny, Miller, & Kudva, 1993).

The works belonging to the first category aim the optimal design of a batch processing plant subject to scheduling restrictions. According to the researchers, scheduling can be considered during the design phase and the sizes and types of equipment may be determined more efficiently (i.e. cheaper). In this approach, the objective of scheduling is either to decrease the production cycle-time (overall completion time) of a product (Voudouris & Grossmann, 1996), or group of products (Vaselenak et al., 1987), or form campaigns (Cerda et al., 1989Coulman, 1989) or minimize idle time rather than to schedule daily operations with respect to the changing market conditions (Birewar & Grossmann, 1990a). In these works, a unified approach applicable to all batch production schemes is not present. The suggested models assume a priori the availability of resources and the stability of market demand structure. Thus, the design attained may not be optimum under varying conditions encountered in daily operations.

The second category of publications focuses on the scheduling of existing batch production plants under variable market conditions. Most of these works have been developed for rather limited types of production schemes. While some of the works deal with serial production (Ku & Karimi, 1988, 1990, 1991a; Cao & Bedworth, 1992), some deal with parallel production with identical parallel operators (Sawik, 1988; Kusiak, 1990; Musier & Evans, 1991). On the other hand, most of the published work is limited to multi-product plants (Ku & Karimi, 1988; Birewar & Grossmann, 1989, 1990b; Ku & Karimi, 1990, 1991a; Cao & Bedworth, 1992). In addition, majority of the existing works concentrate on sequencing rather than scheduling of batches. In sequencing, time is not of interest and solution procedures do not have to deal with the complexity introduced because of time restrictions. In other words, in sequencing the order of the batches to be produced, is determined rather than determining the complete timetable of the production (Karimi & Hong-Ming, 1988; Wellons & Reklaitis, 1991a,bPekny et al., 1993). On the other hand, almost all of the available literature dealing with time, uses discrete time intervals (Sahinidis & Grossmann, 1991; Reklaitis & Mockus, 1995) for simplification and do not consider the realistic situation where time is continuous. There are very recent works considering time as pseudo-continuous (Pinto & Grossmann, 1995; Mockus & Reklaitis, 1996) and continuous (Ierapetritou & Floudas, 1998a,b, 1999). However, they are either heuristic methods or models which are incapable to handle both multi-product and multi-purpose batch plants with identical and nonidentical parallel operators (machines) like other works of this category.

In most of the suggested scheduling procedures, the objective is to decrease the completion time of a product (cycle time; Kim, Jung, & Lee, 1996) or the idle-times of the operators. Minimization of a cost function, or maximization of a profit function, is rarely the goal (Birewar & Grossmann, 1989Huisman et al., 1990Patsidou & Kantor, 1991a; Weeda, 1992).