Int. J. Production Economics 118 (2009) 118–130

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Two-period production planning and inventory control

Ali Cheaitou [a,*], Christian van Delft [b], Yves Dallery [a,b], Zied Jemai [a]

[a] Laboratoire Génie Industriel, Ecole Centrale Paris, Grande Voie des Vignes, 92295 Chatenay-Malabry Cedex, France
[b] HEC School of Management, Paris (GREGHEC), 78351 Jouy-en-Josas Cedex, France

ARTICLE INFO

Available online 23 August 2008

Keywords:
Stochastic production and inventory planning
Capacity constraints
Salvage opportunities
Dynamic programming

ABSTRACT

We study a single product two-period production/inventory model, in which the demands at each period are independent random variables. To optimally satisfy these random demands, quantities can be produced at the beginning of each period using slow or fast production mode, under capacity constraints. In addition to the usual decision variables for such models, we consider that a certain quantity can be salvaged at the beginning of each period. Such salvage processes are useful if the initial inventory of a period is considered to be too high. The unsatisfied demands for each period are backlogged to be satisfied during the next periods. After the end of the second period, a last quantity is produced in order to satisfy remaining orders and to avoid lost sales. The remaining inventory, if any, is salvaged. We formulate this model using a dynamic programming approach. We prove the concavity of the global objective function and we establish the closed-form expression of the second period optimal policy. Then, via a numerical solution approach, we solve the first period problem and exhibit the structure of the corresponding optimal policy. We provide insights, via numerical examples, that characterize the basic properties of our model and the effect of some significant parameters such as costs, demand variabilities or capacity constraints.

1. Introduction

In this paper, we study style-goods type products, characterized by a short life cycle with uncertain future demands. In the literature, the associate production/inventory management issues are modelled, and analyzed, via the so-called *newsboy* model. This basic model consists in producing/ordering a certain quantity of a given product at the beginning of a unique selling period. As the decision maker has no additional replenishment opportunities, if the ordered quantity is lower than the observed demand, excess demand is lost; on the other hand, if this quantity exceeds the demand, the decider will have to salvage the surplus at the end of the horizon, at a salvage value generally lower than the production cost. The literature about style-goods production and inventory problems, and about the newsboy model and its extensions, is considerable (see Khouja, 1999 for an exhaustive review).

However, in many cases a multi-periodic structure underlies production/inventory management problems. This is well known for long life cycle products (see Vollmann et al., 1988), but even for short life cycle products, as demonstrated by several recent research studies and successful applications (see Fisher and Raman, 1996, Fisher et al., 2001). Such multi-periodic decision processes exhibit an important additional feature with respect to the classical one-period newsboy model: it permits one to be reactive and to adapt the successive orders to the successively observed demand fluctuations. In other words, in a single period model the unique order is issued once, before information about the effective demand is available. On the contrary, in a multi-period model, after each order the realized corresponding

* Corresponding author.
E-mail addresses: ali.cheaitou@graduates.centraliens.net (A. Cheaitou), vandelft@hec.fr (C. van Delft), yves.dallery@ecp.fr (Y. Dallery), zied.jemai@ecp.fr (Z. Jemai).

0925-5273/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2008.08.031

demand can be observed and future orders will clearly exploit this information.

We choose to consider here a *two-period* model. Our results can clearly be seen as building blocks permitting the analysis the structure of optimal decisions in general multi-period decision processes. In addition, we assume that the order sizes are limited by capacity constraints. These constraints could prevent the decision maker from satisfying the random future demand, inducing eventual lost-sales, or even penalties.

Several authors have investigated two-period production models. First, Hillier and Lieberman (2001) analyzed a two-period model with uniformly distributed independent demands. Via a dynamic programming approach, these authors analytically solved this model and, under some simplifying assumptions, proposed an explicit optimal order-up-to-level policy. Lau and Lau (1997, 1998) developed a lost sales two-period model numerically solved via a dynamic programming formulation. Bradford and Sugrue (1990) proposed another class of models in which the second period demand is correlated to the first period demand. A Bayesian update for the second period demand forecast can thus be used after having observed the value of the first period demand. These authors determined a conditional order-up-to-level policy for the second period and an optimal order quantity for the first period. Another important two-period model has been proposed by Fisher and Raman (1996), in which the demand for the whole horizon and the demand for the first period are characterized via a joint probability density function (PDF). Furthermore, the order size for the second period is constrained by a limited amount. The optimal policy is numerically approximated via some efficient heuristics. Gurnani and Tang (1999) considered a two-period model with no demand at the first period. The dynamic structure concerns information available for the sequential decisions: at the end of the first period, exogenous information is collected, permitting one to update the initial forecast for the second period demand. Choi et al. (2003) proposed a quite similar two-stage model with an update of the forecast of the second-period demand via some market information.

Several two-period models including capacity constraints have been developed in the framework of supply contracts. Eppen and Iyer (1997) studied a two-period lost sales model with backup agreement contracts in a forecast-update environment. In this type of contract, the quantity ordered in the second period is constrained and depends on the quantity ordered in the first period. Bassok and Anupindi (1997) studied supply contracts models with minimum commitment: the ordered quantity for the entire horizon has to be greater than an initially fixed commitment. Via a classical dynamic programming approach, these authors have exhibited the structure of the optimal policy for this particular multi-period inventory model with backlogs. Donohue (2000) applied an approach similar to that of Choi et al. (2003) for a model with two production modes. The analysis is focused on return option in order to achieve channel coordination. Sethi et al. (2005) analyzed, in a dynamic programming setting, a class of two-stage quantity flexibility contracts. In these contracts, one can order a first quantity before accurate forecasts are available, then after the demand forecast updates are performed, one can order a second constrained quantity and a third quantity from a spot market that is unconstrained. In Barnes-Schuster et al. (2002), supply contracts with options are investigated. Their model is a two-period one with conditional demand distributions. The model is analyzed from the buyer and supplier points of view. Again, the theoretical analysis is mainly concerned by the channel coordination issue.

In the present paper, we consider a two-period production/inventory model with backlogs. The induced costs are purchasing costs, inventory holding costs and backorder costs. The demands at the first and second period are described by independent random variables, with known probability distributions. We assume that at the end of the second period, the remaining inventory can be sold to a specific market with a given salvage value.

In addition to these classical parameters, we suppose that some preliminary fixed orders are to be delivered at each period. We suppose also that at the beginning of the first period, the initial inventory level can be given (as different from zero). This initial inventory could, for example, result from previous selling seasons, or from preliminary (early) orders.

The proposed model includes several production/ordering modes, with different delivery lead-times, in this way providing more flexibility to the decision maker.

Furthermore, in this model we consider capacity constraints: for each of the quantities ordered during the first and second periods, there is a specific bound that cannot be exceeded.

An important feature of the proposed model is worth being highlighted: at each period, the decision maker has the opportunity of salvaging a part of current inventory. We furthermore assume that the periodic salvage values are greater than the salvage value at the end of the last period. This general salvage process corresponds to several practical cases. First, when a parallel market exists, this market can be considered as a client that buys the products at a price lower than the usual market price. A second case can occur in the framework of a buyer–supplier contract in which a fraction of the orders can be returned to the supplier if the current inventory is considered to be too high with respect to expected future demands. Clearly, in such settings the return price can be lower than the production/ordering cost.

In summary, the model studied in the present paper has the following features:

- first, the periodic ordering process is quite general in the sense that at each time period orders can be made for the different subsequent periods, possibly with different costs and for general demand distributions;
- second, the periodic selling process is quite general, in the sense that, in addition to the classical selling process, it is possible, at the beginning of each period, to sell a part of the available inventory to a parallel market, at a given salvage value;