Computers and Chemical Engineering 33 (2009) 2159–2163

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

An integrated production planning model with load-dependent lead-times and safety stocks

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ARTICLE INFO

Article history:
Received 15 December 2008
Received in revised form 1 June 2009
Accepted 16 July 2009
Available online 28 July 2009

Keywords:
Capacity planning
Clearing function
Safety stock
Load-dependent lead-times
Linear programming

ABSTRACT

The divergence over the years of research paradigms addressing the production planning problem has led to the development of an extensive set of techniques, each of which can address a particular aspect of the practical problem and none of which provides a complete solution. In particular, most approaches fail to address the circular, non-linear dependency between resource utilization, lead-times and safety stocks. We present a non-linear programming formulation of the integrated problem using clearing functions that determines a work release schedule guaranteeing a specified service level in the face of stochastic demand. We introduce an iterative heuristic solution procedure that solves a relaxed LP approximation of the original NLP at each iteration to determine the lead-time profile to set safety-stock levels. Computational experiments suggest that our proposed iterative procedure performs well relative to conventional LP models that assume fixed, workload-independent lead-times.

1. Introduction

Since the late 1950s, a wide variety of optimization models have been applied to problems of supply chain planning and control. As the years have gone by, three distinct paradigms have emerged which have become largely independent of each other, presumably due to the quite different mathematical tools they require. *Mathematical programming models* generally seek a minimum cost or maximum profit allocation of production resources to products over time. One approach, widely applied in the process industries, generates aggregate production plans from optimal short-term schedules constrained by highly detailed resource and process limitations. A second approach, more common in the discrete manufacturing industries, takes an aggregate view of the problem, dividing the planning horizon into discrete time buckets and determining a production rate for each product at each resource in each period (Johnson & Montgomery, 1974; Voss & Woodruff, 2003).

Although both approaches are based on discrete time buckets, they use them in different ways. In the process systems engineering formulations, the time bucket is defined to be much smaller than the smallest processing time (raw processing time), in a manner analogous to numerical integration, justifying the assumption of linearity within a time bucket, as in system dynamics approaches (Sterman, 2000). In the industrial engineering literature, where discrete manufacturing systems dominate, the time bucket is much larger than the raw processing time. Thus many events such as job completions occur within a time bucket, and average values provide a good approximation for the behavior of the system being studied. Both these approaches have their disadvantages. While the industrial engineering models with large time buckets often fail to capture the dynamics of the production system, the process engineering models with small time buckets suffer from the curse of dimensionality due to the size of the resulting formulations. In both approaches, the models are primarily deterministic in nature, and do not explicitly consider the stochastic nature of either the production system or the demand it faces.

Queueing models (e.g., Buzacott & Shanthikumar, 1993; Hopp & Spearman, 2001), on the other hand, explicitly model the stochastic nature of the production process and the material flow between stages of the system. These models capture the non-linear relationships between key system parameters and performance measures, such as utilization and cycle time, effectively. However, they are primarily descriptive in nature, and their emphasis on steady-state solutions limits their usefulness for situations where the demand is a non-time-stationary stochastic process, which is generally the case in industrial practice.

A third stream of research, inventory theory (Hadley & Whitin, 1963; Zipkin, 1997), has developed a rich body of work. The dominant focus here has been on the uncertainty of demand, with

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doi:10.1016/j.compchemeng.2009.07.010

very simple models of capacity being used to represent the production process. This work has resulted in the development of robust inventory management policies such as the base stock policy that can provide useful insights into how to address the consequences of uncertain demand. However, this work has also concentrated primarily on time-stationary systems, and almost exclusively addresses long-run expected behavior.

The result of this segregation of research paradigms has been the development of an extensive set of techniques, each of which can address a particular aspect of the problem faced in practice and none of which can provide a complete solution. In particular, almost all prior approaches fail to address the circular, non-linear dependence between plant loading/utilization, lead-time and safety stocks. Jobs are released into the plant to meet demand, increasing resource utilization and hence the lead-time (the time elapsing between the release of a job into the facility and its completion) in a highly non-linear manner. This, together with demand uncertainty, introduces the need to hold safety stock to satisfy a desired customer service level, requiring additional releases that further increase utilization. In this paper we present a non-linear programming formulation of the complete problem using clearing functions and introduce an iterative heuristic that solves a relaxed LP approximation of the original NLP at each iteration to determine a load-dependent lead-time profile to set the safety-stock limits. We present computational experiments that evaluate the performance of the proposed heuristic, and show that it performs favorably compared to conventional linear programming models with a fixed, exogenous lead-time. Thus the contribution of this paper is to extend previous work using clearing functions for production planning in a deterministic environment (Asmundsson, Rardin, Turkseven, & Uzsoy, 2009; Asmundsson, Rardin, & Uzsoy, 2006) to an environment with stochastic demand, where modelling the relationship between safety stocks, lead-times and resource utilization is critical.

2. Overview of clearing function formulation

In recent years a number of researchers have suggested production planning models that account for the non-linear dependence between workload and lead-times (Missbauer and Uzsoy, in press; Pahl, Voss, & Woodruff, 2005; Pahl, Voss, & Woodruff, 2007). One of the mechanisms to capture the non-linear dependence between utilization and throughput in an aggregate level is the clearing function (Karmarkar, 1989). As depicted in Fig. 1, the workload (sum of work in progress and releases at a given period) places an upper bound on the throughput. The expected throughput in a given planning period is a non-decreasing concave function of the workload.

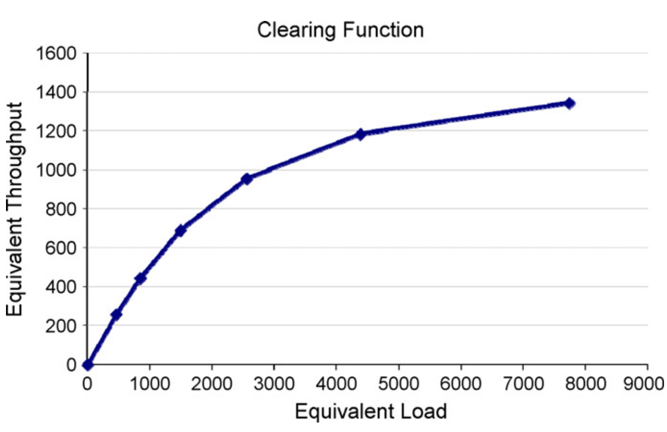

Fig. 1. Clearing function where equivalent load is sum of equivalent throughput (a function of equivalent work-in-progress) and equivalent work-in-progress.

Model

Objective Function

Min TC

Subject to

Cost Function

$TC=\Sigma_t c_w W_t + c_I I_t + c_r R_t + c_s S_t$

WIP Balance

$W_t = W_{t-1} + R_t - X_t$

Inventory Balance

$I_t = I_{t-1} + X_t - D_t + S_t$

Capacity Limit

$X_t \le \alpha_i (X_t + W_t) + \beta_i$

Inventory Limit

(1) $z\,(\Sigma_{\tau=t}^{t+LT_\tau-1} \sigma_\tau^2)^{0.5} + \Sigma_{\tau=t}^{t+LT_\tau-1} D_{mean,\tau} \le W_t + I_t$

(2) $z\,(\Sigma_{\tau}^{t+LT_\tau-1} \sigma_\tau^2)^{0.5} + \Sigma_{\tau}^{t+LT_\tau-1} D_{mean,\tau} \le \Sigma_t R_t$

(3) $z\,(\Sigma_{\tau}^{t} \sigma_\tau^2)^{0.5} + \Sigma_{\tau}^{t} D_{mean,\tau} \le \Sigma_t X_t$

Nomenclature

c_w: Cost of holding unit work in progress inventory
c_I: Cost of holding unit finished goods inventory
c_r: Unit production cost
c_s: Penalty cost of unit shortage
W_t: Work in progress inventory at end of period t
I_t: Finished goods inventory at end of period t
R_t: Releases in time period t
S_t: Shortage at end of period t
X_t: Throughput in period t
D_t: Demand in period t
LT_t: Mean Lead Time in period t
TC: Total Cost
$D_{mean,t}$: Mean demand in period t
σ_t : Variance of demand in period t
β_i: y-intercept of piecewise linear segment i of clearing function
α_i: slope of piecewise linear segment i of clearing function (capacity)

Fig. 2. LP model with clearing function, with decision variables LT_t, W_t, X_t, I_t, R_t, S_t and TC.

The form of this relationship has been verified both through analytical study of queueing models (Asmundsson et al., 2009; Selçuk, Fransoo, & de Kok, 2007) and extensive empirical evidence using simulation models (Asmundsson et al., 2006, 2009). An advantage of the clearing function approach is that it relates the state of the production system to its performance in a memory-less fashion, requiring only knowledge of the current work load. Furthermore, the linear programming structure is preserved even after piecewise linearization (Asmundsson et al., 2006, 2009). Fig. 2 summarizes this LP formulation in relation to our work. Extensive discussion of the clearing function concept and related optimization models is given in Missbauer and Uzsoy (in press). Extensive computational experiments have shown the promising performance of clearing function based production planning models in the presence of deterministic demand.

A common approach to setting safety-stock levels in practice is establishing a base stock level that assumes the distribution of demand over the lead-time to be normal (e.g., Eppen and Martin, 1988). Let us denote the mean and standard deviation of the replenishment lead-time by μ_L and σ_L, respectively, and let μ_D and σ_D denote the mean and standard deviation of the demand per period. Then the base stock level can be established as

$$Y = \mu_L \mu_D + z_\alpha \sqrt{\mu_L \sigma_D^2 + \sigma_L^2 \mu_D^2} \quad (1)$$

where z_α denotes the number of standard deviations required to provide a service level of α, or, equivalently, a stock-out probability of $1-\alpha$. Eppen and Martin (1988) also discuss the performance of this approach when demand over lead-time violates the assumption of normality.