Computers and Chemical Engineering 35 (2011) 2929–2944

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Resource-constrained production planning in semicontinuous food industries

Georgios M. Kopanos[a], Luis Puigjaner[a], Michael C. Georgiadis[b],*

[a] Department of Chemical Engineering, Universitat Politècnica de Catalunya, ETSEIB, Av. Diagonal 647, 08028 Barcelona, Spain
[b] Department of Chemical Engineering, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

ARTICLE INFO

Article history:
Received 26 January 2011
Accepted 18 April 2011
Available online 14 May 2011

Keywords:
Production planning
Resource-constrained planning
Re-planning
MILP
Food industry
Dairy industry

ABSTRACT

The resource-constrained production planning problem in semicontinuous multiproduct food industries is addressed. In particular, the case of yogurt production, a representative food process, in a real-life dairy facility is studied in detail. The problem in question is mainly focused on the packing stage, whereas timing and capacity constraints are imposed with respect to the batch stage to ensure the generation of feasible production plans. A novel mixed discrete/continuous-time mixed-integer linear programming model, based on the definition of families of products, is proposed. Timing and sequencing decisions are taken for product families rather than for products; thus, reducing significantly the model size. Additionally, material balances are realized for every particular product, permitting the detailed optimization of inventory and operating costs. Packing units operate in parallel and share resources. Qualitative as well as quantitative objectives are considered. Several industrial case studies, including also some unexpected events scenarios, have been solved to optimality.

1. Introduction

The theme of production planning for the process industries has received significant attention in the past 20 years. Initially, from the early 1990s to the early 2000s, this was due to the resurgence in interest in flexible processing either as a means of ensuring responsiveness or adapting to the trends in process industries towards lower volume, higher value-added materials in the developed economies (Shah, 1998). More recently, the topic has received a new impetus as enterprises attempt to optimize their overall supply chains in response to competitive pressures or to take advantage of recent relaxations in restrictions on global trade.

The production planning problem at a single site is usually concerned with meeting fairly specific production requirements. Customer orders, stock imperatives or higher-level supply chain or long-term planning would usually set these. It is concerned with the allocation over time of scarce resources between competing activities to meet these requirements in an efficient fashion. The key components of the resulting resource-constrained planning problem are resources, tasks and time. The resources need not be limited to processing equipment items, but may include material storage equipment, transportation equipment (intra- and inter-plant), operators, utilities (e.g., steam, electricity, and cooling water), auxiliary devices and so on. The tasks typically comprise processing operations (e.g., reaction, separation, blending, and packing) as well as other activities which change the nature of materials, and other resources such as transportation, quality control, cleaning, and changeovers. There are both external and internal elements to the time component. The external element arises out of the need to co-ordinate manufacturing and inventory with expected product liftings or demands, as well as scheduled raw material receipts and even service outages. The internal element relates to executing the tasks in an appropriate sequence and at right times, taking account of the external time events and resource availabilities. Overall, this arrangement of tasks over time and the assignment of appropriate resources to the tasks in a resource-constrained framework must be performed in an efficient fashion, which implies the optimization, as far as possible, of some objective. Typical objectives include the minimization of total cost or maximization of profit, maximization of customer satisfaction, minimization of deviation from target performance (Shah, 1998).

Mathematical programming techniques, especially Mixed-Integer Linear Programming (MILP) because of its rigorousness, flexibility and extensive modeling capability, have become one of the most widely explored methods for process planning and scheduling problems (Floudas & Lin, 2005). The application of mathematical programming approaches implies the development of a mathematical model and an optimization algorithm. Most approaches aim to develop models that are of a standard form (from linear programming models for refinery planning to mixed-integer non-linear programming models for multipurpose batch plant scheduling). These may then be solved by standard software or specialized algorithms that take account of the problem structure. A critical feature of mathematical programming approaches

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* Corresponding author. Tel.: +30 2310994184
E-mail address: mgeorg@otenet.gr (M.C. Georgiadis).

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doi:10.1016/j.compchemeng.2011.04.012

Nomenclature

Indices/sets

$f, f' \in F$	product families (families)
$j, j' \in J$	processing units (units)
$k \in K$	renewable resources
$n \in N$	planning time periods
$p \in P$	products
$r \in R$	batch recipes (recipes)

Subsets

F_j	families f that can be processed in unit j
F_k	families f that share the same renewable resource k
F_r	families f that have the same recipe origin r
J_f	available units j to process family f
J_p	units j that can process product p
P_f	products p that belong to the same family f
P_r	products p that have the same recipe origin r
R_f	recipe origin r for family f
R_j	recipes r that can be processed in unit j
R_p	product p that comes from recipe r

Parameters

α_{jn}	daily opening setup time for every unit j in period n (e.g., accounts for the pasteurization and homogenization stages)
β_{jn}	daily shutdown time for every unit j in period n (e.g., cleaning of yogurt production line for hygienic and quality reasons)
$\gamma_{ff'j}$	changeover time between family f and f' in unit j (e.g., accounts for cleaning and sterilizing operations)
δ_{pj}	setup time for product p on unit j
ε_{kfj}	renewable resource k requirements for family f when processed in unit j; in the current study corresponds to the number of workers
E_{kn}^{max}	maximum total capacity of renewable resource k at period n
ζ_{pn}	production target for product p in period n
ζ_{pn}^{cup}	production target for product p in period n (in cups)
η_p^{cup}	cup weight for product p
θ_{pjn}	variable operating cost for processing product p in unit j in period n (e.g., includes labor and utilities costs)
λ	a very small number (0.001)
Λ_{jn}	$= \omega_{jn} - \alpha_{jn} - \beta_{jn}$
M_{jn}	$= \omega_{jn} - \beta_{jn}$
μ_{rn}^{max}	maximum production capacity of recipe r in period n
$\mu_{rn}{}^{min}$	minimum produced quantity of recipe r in period n (e.g., accounts for pasteurization and fermentation tanks capacity restrictions)
ν_{jn}	fixed cost for utilizing processing unit j in period n
ξ_{pn}	inventory cost for product p at time n
o_{jn}	additional unit preparation time for processing unit j in period n
π_{pjn}^{max}	maximum production run for product p in unit j in period n
π_{pjn}^{min}	minimum production run for product p in unit j in period n
ρ_{pj}	processing rate for product p in unit $j \in J_p$
σ_{rn}	release time for recipe r in period n
τ_r	minimum time for preparing recipe r (e.g., for producing stirred yogurt products stands for the minimum fermentation time, while for set yogurt products reflects the minimum cooling time before the packing stage)
$\phi_{ff'jn}$	changeover cost between family f and f' in unit j in period n (e.g., accounts for cleaning and sterilizing operations)
χ_{rn}	cost for producing recipe r in period n
ψ_{pn}	external production penalty cost for product p in period n
ω_{jn}	physical available processing time in unit j at period n

Continuous variables

C_{fjn}	completion time for family f in unit j in period n
I_{pn}	inventory of product p at time n
Q_{pjn}	produced amount of product p in unit j in period n
Q_{pn}^{ext}	external production of product p in period n
Q_{pn}^{int}	total internal production of product p in period n
S_{fjn}	starting time for family f in unit j in period n
T_{fjn}	processing time for family f in unit j in period n

Binary variables

V_{jn}	1 if unit j is used in period n
$W_{f'j'fjn}$	1 if family f', assigned to unit j' in period n, is overlapped by family f, assigned to unit $j \neq j'$ in the same period n
$X_{ff'jn}$	1 if family f is processed exactly before family f', when both are assigned to the same unit j in the same period n
$\bar{X}_{f'j'fjn}$	1 if family f', assigned to unit j' in period n, starts processing before family f, assigned to unit $j \neq j'$ in the same period n
Y_{fjn}	1 if family f is assigned to unit j in period n
$\bar{Y}_{pjn}$	1 if product p is assigned to unit j in period n
Y_{rn}^{R}	1 if batch recipe r is produced in period n
$Z_{f'j'fjn}$	1 if family f', assigned to unit j' in period n, is completed after starting family f, assigned to unit $j \neq j'$ in the same period n

is the representation of the time horizon. This is because activities interact through the use of resources and therefore the discontinuities in the overall resource utilization profiles must be tracked over time; to be compared with resource availabilities to ensure feasibility. The complexity arises because these discontinuities (unlike discontinuities in availabilities) are functions of any schedule proposed and are not known in advance. Excellent reviews on the optimal scheduling for the process industries can be found in Kallrath (2002) and Méndez, Cerdá, Grossmann, Harjunkoski, and Fahl (2006).

The literature in the field of production scheduling and planning of food processing industries is rather poor. Entrup, Günther, Van Beek, Grunow, and Seiler (2005) presented three different MILP model formulations, which employ a combination of a discrete and a continuous time representation, for scheduling and planning problems in the packing stage of stirred yogurt production. They accounted for shelf life issues and fermentation capacity limitations. However, product changeover times and production costs were ignored. The latter makes the proposed models more appropriate to cope with planning rather than scheduling problems, where products changeovers details are crucial. The data set