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Production planning in data envelopment analysis

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ABSTRACT

The present paper extends prior researches on data envelopment (DEA)-based production planning in a centralized decision-making environment. In such an environment, the production planning problem involves the participation of all individual units, each contributing in part to the total production. The production planning problem involves determining the number of products to be produced by all individual units in the next season when demand changes can be forecasted. The current study is concerned with optimal production planning in a centralized decision-making environment. The approach proposed in this paper takes the size of operational units into consideration and the production level for each unit becomes proportional to the ability of the units. The applicability of the proposed approach in real applications is illustrated empirically using two real cases.

1. Introduction

Data envelopment analysis (DEA) was introduced in 1978 when Charnes, Cooper and Rhodes (CCR approach) demonstrated how to change a fractional linear measure of efficiency into a linear programming format. In DEA, decision making units (DMUs) could be assessed on the basis of multiple inputs and outputs. Since the first DEA model developed, many other DEA models and applications have been developed and extended (see Cooper et al. (2004); Amirteimoori and Emrouznejad (2011); Amirteimoori (in press)).

An important application of DEA, from both a practical organizational standpoint and a cost research perspective, is the problem of production planning. The production planning problem is common to many production systems and typically implies assigning required products to the available resources. Many contributions to this theory have been made in the last two decades from various perspectives; see, for instances, Kim and Kim (2001), Sharma (2007a,b), Chazal et al. (2008), Pastor et al. (2009), Sharma (2008) and Sharma (2009a,b).

Kim and Kim (2001) proposed an iterative approach to finding the capacity-feasible production plan. An extended formulation of the LP model was proposed, in order to consider the workload profile of the production quantity and the actual amount of the capacity to be allocated to the requirements for each producer.

Chazal et al. (2008) studied the production planning and inventory management problem based on the assumption that the firm under consideration performs in continuous time on a finite period in order to dynamically maximize its instantaneous profit. Pastor et al. (2009) presented a case of production planning in a woodturning company, where it met the demand at a minimum cost while being subjected to a series of principal conditions.

Using the DEA technique in production planning is not new. As far as we are aware, four DEA-based approaches have been published in the literature: Golany (1988), Beasley (2003), Korhonen and Syrjanen (2004) and Du et al. (2010). Golany (1988), for instance, presented an interactive linear programming procedure to set up goals for desired outputs. Their procedure is based on the empirical production functions generated by DEA and is then adjusted by new information provided by the decision maker in each iteration.

Beasley (2003) proposed nonlinear resource allocation models to jointly decide on the input and output amounts to each DMU for the next period while maximizing the average efficiency of all DMUs. Korhonen and Syrjanen (2004) developed a DEA-based interactive approach to a resource allocation problem that typically appears in a centralized decision making environment. Du et al. (2010) look at the production planning problem, from the productivity and efficiency perspective, using DEA. They have proposed two planning ideas in a centralized decision-making environment when demand changes can be forecasted.

The current paper is concerned with the production planning problem in a centralized decision making environment. It has been assumed that both, supplies for the inputs and demands for the outputs, can be forecast in the next production season. With the forecasted demands for the outputs and supplies for the

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inputs, the paper develops a DEA-based production planning approach to determine the most favorable production plans. We axiomatically assume that output productions can be increased in the next production season when input usages are increased. The approach proposed in this paper takes the size of the operational units into consideration and the production levels for each unit become proportional to the ability of the units. The term "ability" in our study means that the planned production is proportional to the input usages and output productions from a size point of view. In other word, the approach should not set a huge production plan for DMUs with small inputs and outputs. We will define two magnitude sizes for each DMU: a size on the input side and a size on the output side. These definitions will be used to determine a practicable production plan in the next production season.

The rest of the paper is organized as follows: the following section provides a background of the subject. The proposed production planning model is given in Section 3. Then, a simple example is given to illustrate the proposed approach. Section 5 applies the approach to a real data set consisting of 14 Iranian gas companies. Finally, we conclude with the result.

2. Preliminaries

DEA is a mathematical programming model that measures the relative efficiency of operational units with multiple inputs and outputs but with no obvious production function to aggregate the data in its entirety. The principal advantage of the DEA technique is that it does not require the specification of a particular functional form for the technology.

Assume there are n DMUs and the performance of each DMU is characterized by a production process of m inputs (x_{ij}:i=1, …, m) to yield s outputs (y_{rj}:r=1, …, s). Relative efficiency is defined as the ratio of weighted sum of outputs to the weighted sum of inputs. Charnes et al. (1978) proposed measuring the relative efficiencies of a set of n DMUs by solving the following linear fractional program for each DMU_o:

$$\begin{aligned} Max \quad & \theta_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ s.t \quad & \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \le 1, \quad j=1,\ldots,n, \\ & u_r, v_i \ge \varepsilon, \quad \text{for all } r,i, \end{aligned} \tag{1}$$

where u_r and v_i are the factor weights assigned to the rth output and the ith input, respectively. u_r and v_i are the set of most favorable weights for the DMU_o in the sense of maximizing the ratio scale. The objective is to obtain these weights that maximize the efficiency of the unit under evaluation, DMU_o. $\varepsilon > 0$ is a non-Archimedean constant defined to be smaller than any positive real number. In model (1) we maximize the efficiency of the DMU_o being considered, under the conditions that the efficiencies of all DMUs with respect to the weights chosen for DMU_o lie between zero and one. This linear fractional program can be converted into a linear program using Charnes and Cooper (1962) transformation as follows:

$$\begin{aligned} Max \quad & \theta_o = \sum_{r=1}^{s} u_r y_{ro} \\ s.t \quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j=1,\ldots,n, \\ & u_r, v_i \ge \varepsilon, \quad \text{for all } r,i. \end{aligned} \tag{2}$$

This model is a constant return to scale program and it assumes that the status of all input/output variables is known, prior to solving the model. The efficiency ratio θ_o ranges between zero and one, with DMU_o being considered relatively efficient if it receives a score of one.

3. Production planning model

In many organizations with a centralized decision making environment, production usually involves the participation of more than one individual unit, each contributing in part to the total production. The planning problem involves determining the number of products to be produced by all individual units in the next season when demand changes can be predicted. We axiomatically assume that output productions can be increased in the next production season when input usages are increased. We assume that there are n DMUs indexed by $DMU_j : (j=1,\ldots,n)$. The ith input and rth output of DMU_j are symbolized by x_{ij}: $(i=1,\ldots,m)$ and $y_{rj} : (r=1,\ldots,s)$, respectively. Suppose that the demand change for output $r : (r=1,\ldots,s)$ in the next production season can be forecast as D_r. Moreover, suppose that the supply change for input $i=1,\ldots,m$ in the next production season can be forecast as C_i. There are no restrictions on D_r and C_i; they can be positive, negative or zero. To meet the supply and demand changes, the central unit will determine the most favorable input–output plans for all DMUs.

We introduce the variables d_{rj}, $r=1,\ldots,s$, $j=1,\ldots,n$ to represent the demand change of output r for DMU_j in the next production season. Therefore, $\bar{y}_{rj} = y_{rj} + d_{rj}$ is the amount of total rth output of DMU_j in the next season. Similarly, we use the variables c_{ij}, $i=1,\ldots,m$, $j=1,\ldots,n$ to represent the supply change of input i for DMU_j in the next season, and hence, $\bar{x}_{ij} = x_{ij} + c_{ij}$ is the amount of ith input of DMU_j in the next production season. Clearly, we must have $\sum_{j=1}^{n} d_{rj} = D_r$ and $\sum_{j=1}^{n} c_{ij} = C_i$ for all r and i, respectively. The main assumption in our analysis is that output productions can be increased in the next production season when input usages are increased. In other words, we assume that the operational units have the power to increase their outputs productions when their inputs usages are increased. In the proposed approach to production planning, we believe that the inputs and outputs in the next season should be changed, such that each DMU_j has an efficiency score greater than or equal to e_j (e_j is the relative efficiency of DMU_j in the current season). Hence, we must have the following:

$$\begin{aligned} & \frac{\sum_{r=1}^{s} u_r (y_{rj} + d_{rj})}{\sum_{i=1}^{m} v_i (x_{ij} + c_{ij})} \ge e_j, \quad j=1,\ldots,n, \\ & \sum_{j=1}^{n} d_{rj} = D_r, \quad r=1,\ldots,s, \\ & \sum_{j=1}^{n} c_{ij} = C_i, \quad i=1,\ldots,m, \\ & u_r, v_i \ge 0, \quad \text{for all } r,i, \\ & \begin{cases} d_{rj} \ge 0 & \text{when } D_r \ge 0, \\ d_{rj} \le 0 & \text{when } D_r \le 0, \\ c_{ij} \ge 0 & \text{when } C_i \ge 0, \\ c_{ij} \le 0 & \text{when } C_i \le 0. \end{cases} \end{aligned} \tag{3}$$

Since u_r, v_i, d_{rj} and c_{ij} are decision variables, this system of equations is clearly nonlinear. If we make the change of variables $u_r d_{rj} = \bar{d}_{rj}$ and $v_i c_{ij} = \bar{c}_{ij}$, then system (3) reduces to the following form:

$$\frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \bar{d}_{rj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{i=1}^{m} \bar{c}_{ij}} \ge e_j, \quad j=1,\ldots,n,$$