Available online at www.sciencedirect.com

SciVerse ScienceDirect

Fuzzy Sets and Systems 206 (2012) 21–38

FUZZY sets and systems

www.elsevier.com/locate/fss

Multi-period Mixed Production Planning with uncertain demands: Fuzzy and interval fuzzy sets approach

Juan Carlos Figueroa-García[a,*], Dusko Kalenatic[b], Cesar Amilcar Lopez-Bello[a,1]

[a] *Universidad Distrital Francisco José de Caldas, Bogotá, Colombia*
[b] *Universidad de la Sabana, Chía, Colombia*

Received 22 April 2010; received in revised form 13 March 2012; accepted 14 March 2012
Available online 23 March 2012

Abstract This paper shows a general model of a Mixed Production Planning problem with fuzzy demands. The main focus is the development of a model for Production Planning using fuzzy sets in order to use classical mathematical programming techniques to reach an optimal solution over a multiple criteria context. The classical Fuzzy Linear Programming model namely the *Soft Constraints Model* is used to involve flexibility in the problem. Moreover, an Interval Fuzzy Set approach is used to involve uncertainty in the problem.

Keywords: Fuzzy linear programming; Production planning; Fuzzy sets; Interval Type-2 fuzzy sets

1. Introduction and motivation

In the last decades, optimization and heuristic techniques have become useful tools for managers and engineers. Different suppositions and theories about uncertainty have supported the implementation of hybrid methods within mathematical programming, heuristics and computational intelligence.

The most commonly used definition of the parameters of a linear programming (*LP*) model is *deterministic*. LP attempts to solve problems with no uncertain or flexible parameters and variables on the basis of both perfect measures and unbiased samples. For instance, all parameters of an LP model are deterministic values, that is:

$$\begin{aligned} &Opt\{c'x + c_0\} \\ &s.t.\ Ax \leq b \\ &x \geq 0 \end{aligned} \tag{1}$$

where $x, c \in \mathbb{R}^n$, $c_0 \in \mathbb{R}$, $b \in \mathbb{R}^m$ and $A_{n\times m} \in \mathbb{R}^{n\times m}$.

* Corresponding author. Laboratory for Automation, Microlectronics and Computational Intelligence (LAMIC). Tel.: +57 3158727432.
E-mail addresses: jcfigueroag@udistrital.edu.co (J.C.Figueroa-García), duskokalenatic@yahoo.com (D. Kalenatic), clopezb@udistrital.edu.co (C.A.Lopez-Bello).
[1] Mathematical Modeling Applied to Industry (MMAI) research group.

0165-0114/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.fss.2012.03.005

Here, all parameters are considered as real numbers with no oscillations, in other words, all parameters are either *deterministic* or *Crisp*. In this way, we use mathematical theory of evidence concepts to define all parameters over both Type-1 fuzzy sets and Interval Type-2 fuzzy sets environments. For simplicity, in this paper we refer to Type-1 fuzzy sets as Fuzzy Sets (FS) and Interval Type-2 fuzzy sets as Interval Fuzzy Sets (IFS).

The main motivation of this paper is to define a general model of the Mixed Production Problem (*MPP*) involving flexible demands by FS as well as uncertain ones by using IFS. Section 1 introduces the topic. In Section 2, a literature review about fuzzy optimization is shown. Section 3 presents the crisp Mixed Production Planning (*MPP*) problem. Section 4 introduces the Flexible MPP problem using fuzzy constraints. Section 5 defines the Uncertain MPP problem with Interval fuzzy constraints. In Section 6, an implementation example is given and finally Section 7 presents the conclusions of the work.

2. Fuzzy linear programming: Bibliographic review

As soon as fuzzy sets and fuzzy logic were proposed by Zadeh in the 1960s, fuzzy inference and modeling became an important field of study for researchers and managers who regarded uncertainty as a non-statistical problem. Great advances on solving complex problems have appeared as an answer to uncertainty's inherent complexity, which increases every time human beings face more complex problems.

In the last 30 years, many developments in the fields of information, uncertainty and decision making based on fuzzy theory have appeared together with the improvement of computing capabilities. Different decision making approaches have used fuzzy theory to solve complex problems, especially the fuzzy optimization field treated by Klir and Yuan [1], Lai and Hwang [2], Kacprzyk and Orlovski [3], Zimmermann [4], and Zimmermann and Fullér [5]. These authors proposed a general framework for modeling and solving mathematical problems with fuzzy parameters, among them the *Soft Constraints model*, which can be solved through α-cuts or satisfaction degree methods. Tanaka et al. [6], Chanas [7], and Carlsson and Korhonen [8] used parametric methods for fuzzy linear programming problems. Some conceptual papers about possibilistic optimization were written by Lodwick and Jamison [9], Lodwick and Bachman [10], Inuiguchi and Sakawa [11,12], Inuiguchi and Ramík [13], and Figueroa and López [14,15]. Some extensions to intuitionistic fuzzy sets have been presented by Angelov [16], Dubey et al. [17], and the first approaches to interval fuzzy linear programming were presented by Figueroa [18–20], Figueroa and Hernández [21].

Fuzzy production planning problems have been addressed by Peidro et al. [22,23], Mula et al. [24], Lee et al. [25], Lee and Kim [26], Chanas et al. [27], Ángeles Gil et al. [28], and Gen et al. [29]. In this paper we address the problem of uncertain demands of a production planning problem with crisp constraints and different fuzzy partial orders $\lesssim$, $\gtrsim$, $\precsim$ and $\succsim$ through two approaches: the proposal of Zimmermann [4] and the IFS model proposed by Figueroa [18–20], Figueroa and Hernández [21].

3. The crisp Mixed Production Planing (MPP) problem

An important activity in Production Planning is the optimization of the production quantities of "j" products. LP models are efficient at solving the problem in terms of the capacities of the system as well as the demand of each product. Time units are widely used to express the capacities of the system. However, different units can be used to measure capacities such as energy, materials, space, money units, etc. All of them are included in a general model where these capacities and demands are expressed as restrictions of an LP model and its solution is found through the optimal mix of products regarding an objective function.

A multiple decision criteria model is defined depending on the goal of the system, with different Objective Functions $f(x_{jk}(\cdot))$, where $x_{jk}(\cdot)$ is the set of all decision variables which compose the goal. In this way, the main MPP model is defined as follows:

$$Opt\ f(x_{jk}(\cdot)) \tag{2}$$

$$s.t.\ \sum_{j=1}^{n} ts_{ijk} x_{jk}^{r} \leq Ac_{ik}^{r} \quad \forall i \in \mathbb{N}_m; k \in \mathbb{N}_K \tag{3}$$