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Error-sensitivity analysis of hourglass worm gearing with spherical meshing elements

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ABSTRACT

This paper aims to study the influence of tooth errors and shaft misalignments on the tooth contact in hourglass worm gearing with spherical meshing elements. The geometries of the worm and worm-gear tooth surfaces are described, and the distribution characteristic of contact lines is explored. Based on the common moving frame of conjugate tooth surfaces, the errors and the variations are quantified, and the error-variation equation is developed; in order to evaluate the influence levels of different errors, the error-sensitivity formulas are deduced and illustrated by a numerical example. The results provide a theoretical basis for the manufacture and the tooth contact analysis of hourglass worm pair with spherical meshing elements.

1. Introduction

Worm gearing is one of most important mechanisms for transmitting the rotation and the torque between spatial crossed axes, and it is widely used in industrial gear systems that demand high transmission ratio, steady working situation and compact structure. There are three different types of worm gearing: (i) non-throated cylindrical worm drive, (ii) single-throated conical worm drive, and (iii) double-throated hourglass worm drive, and their theoretical and technical problems in design, manufacture and analysis have been the subjects of intensive researches of many scholars. In recent years, Fang et al. proposed some tooth profile modification methods for ZE, ZN and ZK-type worm gearings, and studied the effects on the meshing performance of these worm gearings [1–4]. In order to lessen the transmission error in the manufacture and assembly processes of conical worm drive, Litvin and Donno presented a method to modify the tooth surfaces with localized bearing contact [5]. By means of the meshing simulation, Zhang and Xu obtained accurate conditions for the formation of contact envelope and tooth undercutting in conical worm gear drive, which presents a basis for the proper design of worm gear blank and tooth geometry [6]. Due to high load-carrying capacity, hourglass worm gearing draws increasing attention from the researchers. Shi et al. applied the finite element method (FEM) to study the localization of the contact zone in the planar double-enveloping hourglass worm gearing [7]. Wang et al. presented a parameter optimization approach for the non-backlash double-roller enveloping hourglass worm gearing, in which the contact and lubrication performances were taken into consideration [8]. Chen et al. explored the real tooth surface of toroidal worm gearing with spherical meshing elements machined by means of the forming method [9].

Due to the impacts of elastic deformation, manufacture and assembly errors, the transmission error cannot be completely avoided, and then the meshing performance of gear pair will turn faulty. If the transmission error is a continuous linear function, a high level of gear vibration and noise will be caused. In order to reduce the negative influence of errors, a high-order discontinuous function of transmission error is usually chosen to absorb the linear error. Livtin et al. applied a parabolic transmission error function as the basis of tooth profile modification, and related findings for traditional gears have been presented [10–14]. Stadtfeld and Gaiser applied a fourth-order function of transmission error to reduce the gear noise and to increase the gear strength of bevel and hypoid gear sets

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[15]. Wang and Fong presented a synthesis modification methodology for the tooth surfaces of a face-milling spiral bevel gear set by means of a predetermined fourth-order polynomial function of transmission error [16]. Xu et al. discussed the contact problem of conjugate surfaces with the effect of assembly errors, and verified the error compensation property of mismatched teeth [17]. Wu et al. studied the error-sensitivity of the mounting error in the point-contact spiral bevel gear pair [18]. Aiming to different types of cylindrical worm gears, Simon conducted an intensive investigation about the influence of tooth errors and shaft misalignments on loaded tooth contact [19]. All these efforts have contributed significantly to the progress of gear design, manufacture and analysis technologies.

The methods to study the transmission error can be roughly classified into two groups: (1) methods in which the effect of tooth deflection under load is taken into account, and (2) methods which are conducted from the kinematics point of view. In this thesis, hourglass worm gearing with spherical meshing elements is considered under the rigidity condition without taking the load-dependent deformation into account. The geometries and the contact characteristics of conjugate tooth surfaces are described, and the error-variation equation, reflecting the inherent relationships between the errors and the variations, is developed. In order to evaluate the influence levels of different errors, the error-sensitivity formulas are deduced and illustrated by a design example of mismatched hourglass worm pair with spherical meshing elements.

2. Theoretical background

2.1. Basic structure

As shown in Fig. 1, hourglass worm pair with spherical meshing elements is composed of a worm, the steel balls and a worm gear. Thereinto, the steel balls are inserted in the finger-like sustaining sockets which distribute circumferentially on the worm gear, and are used as the worm gear teeth. The worm surface is generated by the enveloping of the steel ball, and on the worm the locus of the center of the steel ball is just the directrix of the worm surface, which is a toroidal helix.

In the mesh zone, the stability of the steel ball is guaranteed by the co-action of the worm and the worm gear, and in the outside of the mesh zone the stability is guaranteed by an additional cage-shaped device around the worm gear (this device is omitted in Fig. 1). When hourglass worm gearing works, two kinematic rolling pairs are formed, i.e. the first pair between the worm and the steel ball and the second pair between the steel ball and the worm gear. Additionally, the worm and worm-gear tooth surfaces are free of the curvature interference, which is beneficial to improve the contact ratio; in the manufacture process of the worm gear, the hob cutter with the complex profile is avoided and thus the machining precision can be ensured.

2.2. Worm surface

For hourglass worm gearing, the reference surface of the worm is a torus generated by revolving a circular-arc generatrix in three dimensional space around the worm axis. This generatrix is a part of the reference circle of the worm gear, and is always coplanar with the worm axis. In the coordinate system $\{O_w;X_wY_wZ_w\}$ attached to the worm, the radius vector equation of the circular-arc generatrix C can be defined by:

$$C: \quad \boldsymbol{\rho} = (A - R\cos\varphi)\mathbf{i}_w - R\sin\varphi\mathbf{k}_w \tag{1}$$

where A is the center distance, R is the reference radius of the worm gear; φ is the rotation angle parameter about the worm gear axis Z_g.

According to the generation process of the torus, the reference surface $S^{(d)}$ of the worm can be defined by:

$$S^{(d)}: \quad \mathbf{P}^{(d)} = B(\lambda)\boldsymbol{\rho} = (A - R\cos\varphi)\mathbf{e}(\lambda) - R\sin\varphi\mathbf{k}_w \tag{2}$$

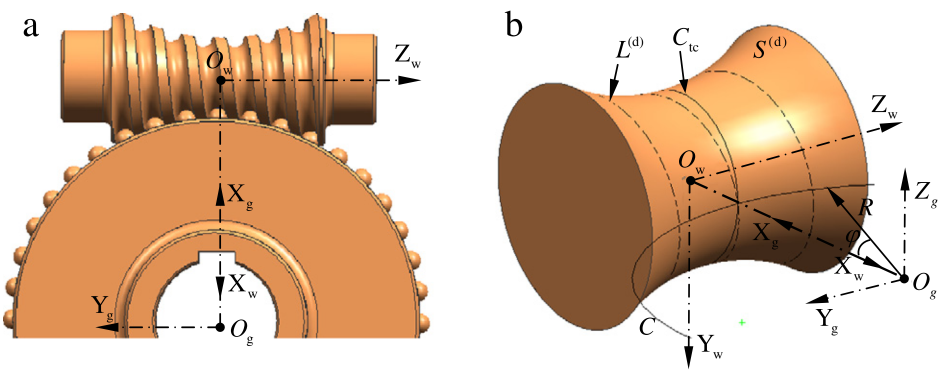

Fig. 1. Hourglass worm pair with spherical meshing elements.