Computers and Structures 129 (2013) 40–53

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

A general material perturbation method using fixed mesh for stress sensitivity analysis and structural shape optimization

Dan Wang, Weihong Zhang *

The Laboratory of Engineering Simulation and Aerospace Computing, Northwestern Polytechnical University, Xi'an 710072, China

ARTICLE INFO

Article history:
Received 12 February 2013
Accepted 18 August 2013
Available online 19 September 2013

Keywords:
Shape optimization
Grid perturbation method
Material perturbation method
Stress sensitivity analysis
Stress correction

ABSTRACT

Stress sensitivity analysis constitutes an essential problem in gradient-based structural shape optimization. Unlike the traditional grid perturbation method (GPM), a general material perturbation method (MPM) using a fixed mesh is originally developed to simplify the sensitivity analysis scheme in this work. A domain function is introduced to characterize the boundary perturbation, whose effect is considered by correcting simultaneously stiffness matrices and stresses of elements attaching the perturbed boundary. Implementations of the MPM on shape optimization of plane stress, axisymmetric, 3D and thin-walled curved shell problems show that the proposed method has the advantage of efficient and explicit computing of stress sensitivities.

1. Introduction

Shape optimization is one of the most challenging problems in structure designs. Many researchers have devoted themselves in the community [1–6]. As the most efficient algorithms, the difficult and essential part of gradient-based algorithms is sensitivity analysis. Usually, it is carried out by using the grid perturbation method (GPM) to obtain the so-called velocity field [7–9] for the determination of modified nodal locations in response to the boundary perturbation. Different approaches such as mesh mapping, Laplacian morphing, boundary nodal perturbation and physical approach using fictive displacement [10–14] were largely utilized to this aim. The underlying assumption is that any boundary perturbation only changes the mesh shape of a domain, i.e. nodal positions, while the material property of each element is firmly attributed to the element mesh and remains unchanged.

To simplify the design and sensitivity analysis procedure as easily as in topology optimization, attempts were made to extend the concept of Fixed Grid (FG) representation [15] into shape optimization problems [16]. As illustrated in Fig. 1, a rectangular base domain is defined to envelop the considered structure. One such base domain is fully discretized into a structured finite element mesh that can be classified into three subsets: elements inside the structure domain, outside the structure domain and crossed by the domain boundary. The mesh is fixed not only at the step of sensitivity analysis but also at all iteration steps. The main concern is about how to deal with the elements crossed by the moving boundaries. Usually, averaged material properties weighted by the area fraction are assigned to the concerned boundary elements and then used to calculate the stiffness matrix. Dunning et al. [17] proposed a weighted least squares method to improve the accuracy of responses with a weighting function based on both area-fraction and the distance of the sampling point to the boundary. As indicated in their work, the Area-fraction weighted Fixed Grid (AFG) approach may cause poor sensitivity computation of the elemental stress and further lead to some unstable problem of optimization convergence. Local stresses thus obtained are poorly approximated even for a much refined mesh [15] so that the accuracy of corresponding stress sensitivities is worse than expected. This is why the FG representation was mainly limited to the minimization of the structural compliance.

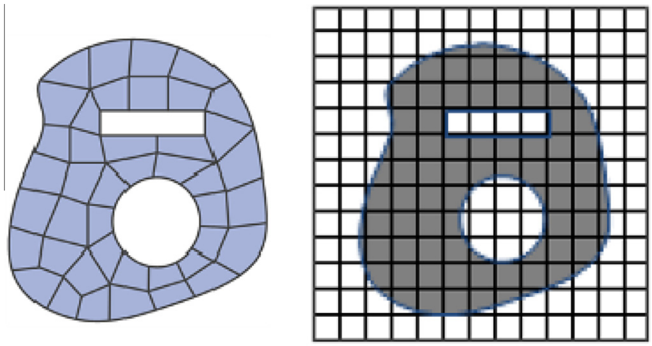
In recent years, stress-based optimization in the framework of a fixed mesh receives much attention. García and Steven improved the stress accuracy greatly [18] by employing a FG global/local analysis, i.e., refining local mesh around the boundary. Kim and Chang [19] studied the stress sensitivities using a fixed mesh as Eulerian representation for shape optimization. The developments of XFEM [20–23] and IGA [4,24] techniques are expected to provide more facilities for shape optimization.

In this paper, the so-called MPM is developed to perform stress sensitivity analysis. The main difference between the proposed method and existing ones is twofold. First, only the structure domain is meshed. The fixed grid approach is only used for sensitivity analysis, while the mesh is updated to track the revised boundary in the usual way during shape optimization iterations. The mesh

* Corresponding author. Address: P.O. Box 552, Northwestern Polytechnical University, 127 You Yi Xi Lu, Xi'an, Shaanxi 710072, China. Tel./fax: +86 (0)29 88495774.

E-mail address: zhangwh@nwpu.edu.cn (W.H. Zhang).

0045-7949/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruc.2013.08.008



(a) (b)

Fig. 1. Distinction between the GPM with a moving mesh and the FG representation with a fixed mesh: (a) the GPM with a moving mesh; (b) the FG reprentation with a fixed mesh.

updating relies upon the preprocessor of the commercial finite element software to make sure that the mesh quality and the FE analysis are qualified for the downstream optimization. This mesh strategy was used in the work of Xia et al. [25] to minimize the structural compliance by changing the layout of components. Second, accuracies of stress sensitivities are improved through stress corrections of specific boundary elements attaching the moving boundary. It is shown that proper corrections of both element stiffness matrices and stress components are essential to achieve a good accuracy of stresses and their sensitivities. A variety of examples are employed to illustrate the MPM.

2. Domain function definition in MPM

For shape optimization of a linearly static structure, it is known that design variables influence the structural responses through the element stiffness matrix given below.

$$\mathbf{K}_i = \int_{\Omega_i} \mathbf{B}_i^T \mathbf{D}_i \mathbf{B}_i d\Omega_i, \tag{1}$$

where $\mathbf{B}_i$ is the strain–displacement matrix only depending upon shape functions of the element and $\mathbf{D}_i$ denotes the elastic matrix of the element depending linearly upon Young's modulus. Clearly, the stiffness matrix of an element depends upon element geometry and assigned material properties.

The MPM interprets the geometric perturbation of the designable boundary as a material shift over a fixed mesh. As shown in Fig. 2(c), only the boundary elements attaching the hole undertake one such material property perturbation. Noticeably, because points of the perturbed shape boundary may be located outside the design domain, a series of imaginary elements, which are not involved in the calculation, are constructed with the same topological structure as the boundary elements in order to describe the case of an enlarged boundary element at the sense of a perturbation.

To calculate the shape sensitivity, the first thing is to investigate how to relate the boundary perturbation to material properties of boundary elements. For an arbitrary boundary element i, its domain function d_i is defined as the space occupied by the material in the element. Practically, the domain function is measured by the material area of a 2D element or the material volume of a 3D element, respectively. It is obviously a function of shape design variables and could be represented as

$$d_i = d_i(\mathbf{r}), \tag{2}$$

where $\mathbf{r}$ denotes the vector of shape design variables.

Suppose the initial mesh fully filled with solid material phase has Young's modulus E and density ρ. Material properties of boundary element i can be approximated in the following way.

$$E_i(\mathbf{r}) = N_i(d_i(\mathbf{r})) \cdot E, \tag{3}$$

$$\rho_i(\mathbf{r}) = N_i(d_i(\mathbf{r})) \cdot \rho, \tag{4}$$

where the interpolation function $N_i(d_i(\mathbf{r}))$ characterizing the effect of the boundary perturbation is defined as the ratio between the domain function of the perturbed element and that of the initial element.

$$N_i(d_i(\mathbf{r})) = \frac{d_i(\mathbf{r})}{d_i^0}, \quad i \in \Omega_b. \tag{5}$$

Ω_b stands for the set of boundary elements. $d_i^0 = d_i(\mathbf{r}^0)$ is the initial area or volume of element i at $\mathbf{r}^0$.

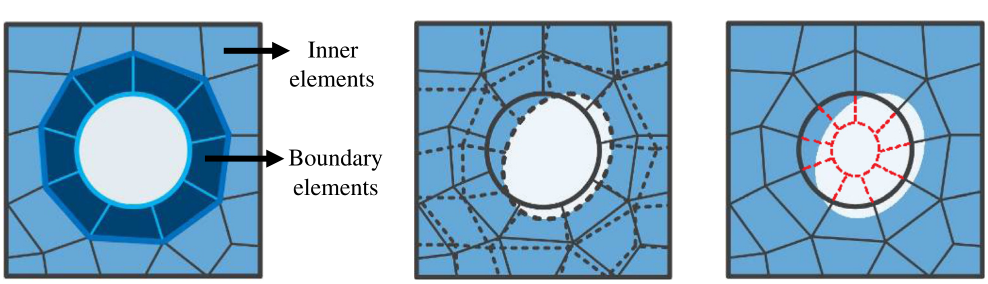
Based on Eqs. (3) and (5), the element stiffness matrix reads

$$\mathbf{K}_i = \int_{\Omega_i} \mathbf{B}_i^T \mathbf{D}_i \mathbf{B}_i d\Omega_i = N_i(d_i(\mathbf{r}))\mathbf{K}_i^0 = \frac{d_i(\mathbf{r})}{d_i^0}\mathbf{K}_i^0, \tag{6}$$

where superscript 0 denotes the initial state at $\mathbf{r}^0$.

It could be concluded from Eq. (6) that the element stiffness matrix depends proportionally upon the elemental space occupied by the material. One such relationship is very similar to the linear expression of the membrane and shear components of the stiffness matrix with respect to the thickness in the optimization of the thickness distribution [26]. Notice that the coupled bending component of the stiffness matrix is a nonlinear function of the thickness, which complicates the process of sensitivity analysis remarkably [27]. Moreover, the thickness usually acts as a kind of design variables in the optimization of the thickness distribution, while the domain function denoting the elemental space occupied by the material is a kind of intermediate variables rather than design variables in the boundary shape optimization by the MPM. Thus, the derivative of the element stiffness could be explicitly expressed as

$$\frac{\partial \mathbf{K}_i}{\partial r_j} = \frac{1}{d_i^0}\frac{\partial d_i}{\partial r_j}\mathbf{K}_i^0, \tag{7}$$



(a) (b) (c)

Fig. 2. Illustrations of different perturbation schemes: (a) the initial mesh; (b) shape perturbation with the GPM; (c) shape perturbation with the MPM.