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Linear statics and free vibration sensitivity analysis of the composite sandwich plates based on a layerwise/solid-element method ☆

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ABSTRACT

Although many researches have been attracted to optimization problems of composite sandwich structures, there are rarely special literatures for sensitivity analysis which provides essential gradient information for the optimization. In this paper the linear statics and free vibration sensitivity analysis problems of the composite sandwich plates are studied based on a layerwise/solid-element method (LW/SE) which was developed in our previous work to eliminate or decrease the error induced by the equivalent methods of the core. In the present sensitivity analysis schemes the cores of the sandwich plates are discretized by three models, namely, full model, local model and equivalent model.

In the numerical examples, two kinds of sensitivity analysis schemes, the overall finite difference method (OFD) and the semi-analytical method (SAM), are employed to calculate the sensitivity coefficients of displacements, stresses and natural frequencies. The convergence is studied together with the effect of step size on the relative error. The performance of these three methods of modeling the honeycomb in computing displacements and natural frequencies sensitivity coefficients is investigated. At last, the influences of the parameters on the displacements, stresses and natural frequencies are investigated by using the sensitivity analysis scheme based on the local model and SAM.

1. Introduction

Sensitivity analysis of structures is a usual approach to obtain the gradient information of the response quantities with respect to the interest design parameters which include intrinsic variables like material properties and thickness, as well as geometric control variables governing the size and shape of the structures. In the past three decades, the sensitivity analysis has evolved as a major research area in structural analysis, holding out immense prospect for widespread applications, for instance, structural optimization, evaluation of structural reliability, and parameter identification. Composite sandwich structures are broadly used in many engineering because they offer a high bending stiffness with the minimum mass, the capability to be tailored, the high damping properties and the great potential for impact protection. Therefore, optimization design of this kind of structures is very important. Although many researchers have been attracted to this branch [1–4], there are rarely investigations for the sensitivity analysis which specially provides the essential gradient information for the optimization.

The past two decades have witnessed a spurt of research activities in the computational aspects of sensitivity analysis, such as the sensitivity analysis of static, eigenvalue, transient response and buckling problems. Generally, the sensitivity analysis consists of variational method and implicit differentiation method [5,6]. The variational methods, which is also referred to as the continuum methods, are based upon differentiating the continuum governing equations of the structural response. Although the continuum method generally used in the shape optimization of the continuous structures is mathematically rigorous, we can hardly use this method due to the difficulties in program coding and application. The implicit differentiation methods, which is also referred to as the discrete methods, are based upon the derivatives of the discrete formulations of the finite element methods or other numerical methods. With the rapid development of the finite element methods, the discrete methods are more and more popular than the continuum methods.

The existing discrete approaches such as analytical method (AM) [7], the overall finite difference methods (OFD) [8] and the semi-analytical method (SAM) [9–18] are commonly used. If the sensitivity analysis is implemented in finite element method (FEM), sensitivity calculations require the derivatives of the stiffness matrixes, the mass matrixes and the load vectors with

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respect to the design variables. In the AM methods, these derivatives are calculated analytically before the evaluation of the sensitivity coefficients. So the AM method provides useful physical insight into the effect of the variation of design or variation of some parameters on the structural response. But it is difficult to calculate the derivatives analytically in many cases, especially for the derivatives with respect to the geometric control variables [19]. The overall finite difference methods, in which the entire analysis is repeated for a perturbed variable, is popular since it is simple and accurate. However, the cost of calculation is very great for large structural systems. As to the semi-analytical approach, the differentiation of the component factors like the stiffness matrix, the load vector and so on is done approximately by finite difference methods, but the final solution procedure follows that of the analytical method. It can be implemented as easily as the OFD method and is as efficient as the SAM method. Thus the semi-analytical method is established based on the advantages of AM and OFD [20]. Obviously, both the OFD and the SA suffer truncation and condition errors which result from the finite difference methods, the magnitude of step size, and the machine accuracy [19].

Recently, the modeling scheme of composite sandwich structures is regarded as following the same analysis schemes of the composite laminated structures, such as the equivalent single layer theory (classical laminate theory and shear deformation laminated plate theories) [21–27], three-dimensional elastic theory (traditional 3-D elastic formulations, layerwise theory, unified formulation and generalized unified formulation) [28–32] and multiple model methods [33]. In the traditional analysis schemes of the composite sandwich structures [34–40], the core is firstly simplified as an equivalent anisotropic material and then modeled by the plates and shells theories. Their main disadvantage is that the equivalent core will result in large equivalent error especially in the key area and the thick core will further reduce the analysis accuracy of the plates and shells theories. For the composite stiffened laminated cylindrical shells, a layerwise/solid-element (LW/SE) method was established based on the layerwise theory and the finite element method (FEM)[41]. And then, for the composite sandwich plates this LW/SE method was extended to eliminate or decrease the error introduced by the equivalent methods about the core [42]. Furthermore, the detailed local deformation of the facesheets and core can be obtained by using this analysis scheme if the core cells belonging to the special attention area (for example, the impact area) are modeled based on the real structure form completely instead of the equivalent form.

In the present work, the linear statics and free vibration sensitivity analysis problems of the composite sandwich plates are studied based on the LW/SE method. Two kinds of sensitivity analysis schemes SAM and OFD are employed to calculate the derivatives of the displacements, the stresses and the natural frequencies.

2. Mathematical formulations

2.1. A brief review of the LW/SE method

The schematic diagram of the LW/SE method for the composite sandwich structures is shown in Fig. 1, where the upper and lower facesheets are discretized with the four-noded quadrilateral elements and the layerwise theory, while the core is discretized with the eight-noded solid elements. Based on the compatibility conditions at the interface between facesheets and core, the layerwise theory can be conveniently coupled with the governing equations of the core established by the brick elements as a result of two characteristics of the layerwise theory. One is that the degree of freedoms (DOFs) of the layerwise theory is equal to that of the brick element, and another is that the displacements variables of the upper and lower surfaces of face sheets appear in the governing equations. Based on the finite element formulations of the facesheets and core, the final governing equation of the composite sandwich structures can be assembled by using the compatibility conditions to ensure the continuity of displacements at the interface between facesheets and core. In the present work, the honeycomb is investigated. Three models, the full model, the local model and the equivalent model, are presented to model the honeycomb. In the full model all details of the honeycomb structures are discretized, as can be seen in Fig. 1b. In the equivalent model the honeycomb is firstly integrally considered as the anisotropic material by using some equivalent theories and then discretized by brick elements as shown in Fig. 1d. Although the equivalent model greatly reduces the computational cost and the difficulty of the algorithm, at the same time, compared to the full model it reduces the analysis accuracy and cannot obtain the detailed local deformation, such as that resulted from the point load and/or point supports. The local model illustrated in Fig. 1c, in which the honeycomb cells in the key region are modeled based on the real micro structure form completely instead of the equivalent anisotropic materials, is a combination of the full model and the equivalent model.

In the layerwise laminate theory[33], the displacements at point (x,y,z) in the composite laminated plates are assumed to be

$$\begin{aligned} u(x,y,z) &= \sum_{i=1}^{N+1} u_i(x,y)\phi_i(z), \\ v(x,y,z) &= \sum_{i=1}^{N+1} v_i(x,y)\phi_i(z), \\ w(x,y,z) &= \sum_{i=1}^{N+1} w_i(x,y)\phi_i(z), \end{aligned} \tag{1}$$

where u, v and w represent the displacement components in the x, y and z directions, respectively. ϕ_i is a linear Lagrangian interpolation function through the thickness of the laminated facesheets of the composite sandwich plates. The laminate thickness dimension is subdivided into a series of N one-dimensional finite elements ($Ne = N + 1$ nodes) whose nodes are located in planes parallel to xy plane in the undeformed laminated facesheets. u_i, v_i and w_i are the nodal values. N is also the number of mathematical layers of the laminated plates, which may be equal to or less than the number of physical layers.

As presented in the previous work, the final discrete equations of the composite sandwich structures are given by

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}, \tag{2}$$

where

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{11}^{t} + \boldsymbol{M}_{11}^{c} & \boldsymbol{M}_{12}^{t} & \boldsymbol{M}_{12}^{c} & \mathbf{0} & \boldsymbol{M}_{13}^{c} \\ \boldsymbol{M}_{21}^{t} & \boldsymbol{M}_{22}^{t} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{M}_{21}^{c} & \mathbf{0} & \boldsymbol{M}_{11}^{b} + \mathbf{M}_{22}^{c} & \boldsymbol{M}_{12}^{b} & \boldsymbol{M}_{23}^{c} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{M}_{21}^{b} & \boldsymbol{M}_{22}^{b} & \mathbf{0} \\ \boldsymbol{M}_{31}^{c} & \mathbf{0} & \boldsymbol{M}_{32}^{c} & \mathbf{0} & \boldsymbol{M}_{33}^{c} \end{bmatrix}, \quad \mathbf{U} = \begin{Bmatrix} \boldsymbol{U}_1^{t} \\ \boldsymbol{U}_2^{t} \\ \boldsymbol{U}_1^{b} \\ \boldsymbol{U}_2^{b} \\ \boldsymbol{U}_3^{c} \end{Bmatrix},$$

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_{11}^{t} + \boldsymbol{K}_{11}^{c} & \boldsymbol{K}_{12}^{t} & \boldsymbol{K}_{12}^{c} & \mathbf{0} & \boldsymbol{K}_{13}^{c} \\ \boldsymbol{K}_{21}^{t} & \boldsymbol{K}_{22}^{t} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{K}_{21}^{c} & \mathbf{0} & \boldsymbol{K}_{11}^{b} + \boldsymbol{K}_{22}^{c} & \boldsymbol{K}_{12}^{b} & \boldsymbol{K}_{23}^{c} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{K}_{21}^{b} & \boldsymbol{K}_{22}^{b} & \mathbf{0} \\ \boldsymbol{K}_{31}^{c} & \mathbf{0} & \boldsymbol{K}_{32}^{c} & \mathbf{0} & \boldsymbol{K}_{33}^{c} \end{bmatrix}, \quad \mathbf{F} = \begin{Bmatrix} \mathbf{0} \\ \boldsymbol{F}_2^{t} \\ \mathbf{0} \\ \boldsymbol{F}_2^{b} \\ \boldsymbol{F}_3^{c} \end{Bmatrix},$$

the superscript t denotes upper facesheet, and the superscript b denotes lower facesheet. For the facesheets, the subscripts 1 and