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Shape sensitivity analysis of stress intensity factors by the scaled boundary finite element method

Morsaleen Shehzad Chowdhury *, Chongmin Song, Wei Gao

Centre for Infrastructure Engineering & Safety, School of Civil & Environmental Engineering, University of New South Wales, Sydney, NSW 2052, Australia

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ABSTRACT

The scaled boundary finite element method (SBFEM) is extended for shape sensitivity analysis of stress intensity factors (SIFs) with respect to the crack geometry. The procedure combines the finite element formulations with the boundary discretization. The original equations of the SBFEM are reformulated as functions of the so-called scaling centre, synonymous to the crack tip. The variation in crack geometry is modelled without remeshing. Sensitivity is analyzed by direct differentiation. Following the computation of the displacement field sensitivity, the SIF sensitivity is evaluated directly from the present SIF definition. Numerical examples are presented.

1. Introduction

Stress intensity factors (SIFs) are used to represent the strength of the singularity near the tip of a crack due to remotely applied loads. They are utilized in failure criterion of cracked structural components. While there are many analytical methods of evaluating the SIFs, like explicit handbook equations (e.g. [1]), recent times have seen numerical techniques to be more favourable due to its computational ease and superiority in handling problems with more complex geometries and boundary conditions. The displacement/stress extrapolation and *J*-integral techniques, based on the finite/boundary element method (FEM/BEM) results, are examples. Some areas of fracture mechanics, however, also require the variation or derivative of the SIFs with respect to the crack geometry, i.e. shape sensitivity. Its significance is well recognized in, for example, the prediction of stability and arrest of a single crack [2,3], universal size effect modelling [4,5] and configurational stability analysis of evolving cracks [6,7]. In particular, shape sensitivity plays a major role in the reliability analysis of cracked structures with uncertainties in the crack geometry [8].

Over the years, various numerical approaches to shape sensitivity analysis have been investigated. The use of predefined equations of the SIFs (e.g. [1]) is an option, but are limited owing to complexities in loading, material behaviour and crack geometry. Direct application of the FEM [9–11] in combination with a finite difference (FD) approximation is also possible, at the computational expense of numerous deterministic analyses and excessive remeshing, especially near the crack tip region. Applying the BEM incurs a simpler boundary mesh [12–14], but still requires remeshing as the crack surface forms part of the boundary. Analytical techniques built on the FEM have also emerged. The virtual crack extension (VCE) technique, proposed by [15], is one of the earliest examples. A fundamental requirement of the technique is mesh perturbation. Hwang et al. [16–18] and Hwang and Ingraffea [19] applied the VCE technique to obtain first/second-order derivatives of multiple crack systems, axisymmetric stress states and crack-face and thermal loading cases. Latter techniques include those based on continuum shape sensitivity theory, which introduces a velocity field expression [20] to simulate the shape variation or

* Corresponding author. Tel.: +61 2 9385 5306; fax: +61 2 9385 6139.
E-mail address: morsaleen.chowdhury@unsw.edu.au (M.S. Chowdhury).

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Nomenclature

a	crack size
A^e	area formed by scaling a boundary element
$\mathbf{B}_1, \mathbf{B}_2$	matrices that describe the strain–displacement relationship
$\mathbf{b}_1, \mathbf{b}_2$	geometry transformation matrices
$\mathbf{c}, \mathbf{c}^s, \bar{\mathbf{c}}^s$	integration constants, singular constants and its normalized form, respectively
$\mathbf{D}$	material elasticity matrix
$\mathbf{E}_0, \mathbf{E}_1, \mathbf{E}_2$	coefficient matrices of the scaled boundary finite element equation
$\mathbf{H}$	anti-symmetric matrix
$\mathbf{J}, J, \mathbf{J}^{-1}$	Jacobian matrix, its determinant and inverse Jacobian matrix, respectively
$K_{\mathrm{I}}, K_{\mathrm{II}}$	mode-I and mode-II stress intensity factors, respectively
$\mathbf{K}, \mathbf{K}_{\mathrm{G}}$	stiffness matrix and global stiffness matrix, respectively
$\mathbf{L}$	linear differential operator
n	number of displacement modes
$\mathbf{N}(\eta)$	shape functions defined in the η direction
O	scaling centre
$\mathbf{Q}$	nodal force vector
r, r_b	radial coordinate and the radial distance to the boundary, respectively
S^e	boundary elements
$\mathbf{u}_b, \mathbf{u}_G$	sub-domain boundary displacements and global displacements, respectively
$\mathbf{u}(\xi)$	displacement functions
$\mathbf{u}(x,y)$, $\mathbf{u}(\xi,\eta)$	Cartesian and scaled boundary displacement fields, respectively
$(x,y), (x_b,y_b), (x_0,y_0)$	domain, boundary and scaling centre coordinates, respectively
$\mathbf{x}_b, \mathbf{y}_b$	vectors containing the Cartesian nodal coordinates
$\mathbf{Z}$	Hamiltonian matrix
β	material orientation
γ	crack orientation or angle
$\mathsf{F}, \mathsf{F}_{\theta_f}, \bar{\mathsf{F}}$	square-root singularity stress modes, the same stress modes interpolated at θ_f and the normalized square-root singularity modes, respectively
δ	direction of change in scaling centre
$\partial/\partial\delta, \partial/\partial a, \partial/\partial\gamma$	sensitivity with respect to δ, a and γ, respectively
$\boldsymbol{\varepsilon}(x,y), \boldsymbol{\varepsilon}(\xi,\eta)$	Cartesian and scaled boundary strain fields, respectively
η	local scaled boundary coordinate
θ, θ_f	polar angular coordinate and the angle ahead of the crack, respectively
λ_i, λ_{Ni}	ith eigenvalue of $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}_N$, respectively
$\boldsymbol{\lambda}, \boldsymbol{\lambda}_N$	eigenvalue matrix and the eigenvalues with negative real parts, respectively
μ	traction factor
ξ	radial scaled boundary coordinate
$\Gamma, \Gamma_t, \Gamma_u$	domain boundary and prescribed surface tractions and displacements, respectively
$\boldsymbol{\sigma}(x,y), \boldsymbol{\sigma}(\xi,\eta), \boldsymbol{\sigma}^s(\xi,\eta), \boldsymbol{\sigma}^s(r,\theta)$	Cartesian and scaled boundary stress fields, and scaled boundary and polar singular stress fields, respectively
$\boldsymbol{\phi}_i, \boldsymbol{\phi}_{ui}, \boldsymbol{\phi}_{\sigma i}$	ith: eigenvector of λ_i, column of $\boldsymbol{\Phi}_u$ and column of $\boldsymbol{\Phi}_\sigma$, respectively
$\boldsymbol{\Phi}, \boldsymbol{\Phi}_u, \boldsymbol{\Phi}_\sigma, \boldsymbol{\Phi}_u^s, \boldsymbol{\Phi}_\sigma^s$	eigenvector matrix, displacement modes and stress modes, and singular displacement modes and stress modes, respectively
$\boldsymbol{\psi}_i$	ith eigenvector of $-\lambda_i$
ω	angular direction of δ
Ω	physically bounded domain

change. Chen et al. [21,22] used this to determine the first-order derivative of the J-integral. Rao and Rahman [23,24] carried on, but also investigated the effects of functionally graded materials. Reddy and Rao [25–27] and Rao and Reddy [28] then developed a continuum shape sensitivity based approach using the well known fractal finite element method (FFEM).

The scaled boundary finite element method (SBFEM) was introduced by [29]. It possesses many characteristics that can simplify shape sensitivity analysis. Unlike the BEM, no fundamental solution is required and unlike the FEM, only the boundary need be meshed. The stress singularity at a crack tip is expressed analytically. Special elements or numerical techniques are not required at the crack tip for fracture analysis, and the crack surface remains meshless. High accuracy and efficiency in evaluating the SIFs have been vastly demonstrated in [30–34]. Only Chowdhury et al. [35,36] have previously explored the possible capabilities of the SBFEM for shape sensitivity analysis.

The purpose of this paper is to continue the works of [35,36] and analytically extend the SBFEM for shape sensitivity analysis of the SIFs. An efficient, accurate and simple procedure is introduced to compute the first-order derivative of the SIFs