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Automatica 38 (2002) 1867–1880

automatica

www.elsevier.com/locate/automatica

Robust optimal design and convergence properties analysis of iterative learning control approaches ☆

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Received 13 June 2001; received in revised form 10 December 2001; accepted 3 June 2002

Abstract

In this paper, we address four major issues in the field of iterative learning control (ILC) theory and design. The first issue is concerned with ILC design in the presence of system interval uncertainties. Targeting at time-optimal (fastest convergence) and robustness properties concurrently, we formulate the ILC design into a min–max optimization problem and provide a systematic solution for linear-type ILC consisting of the first-order and higher-order ILC schemes. Inherently relating to the first issue, the second issue is concerned with the performance evaluation of various ILC schemes. Convergence speed is one of the most important factors in ILC. A learning performance index—Q-factor—is introduced, which provides a rigorous and quantified evaluation criterion for comparing the convergence speed of various ILC schemes. We further explore a key issue: how does the system dynamics affect the learning performance. By associating the time weighted norm with the supreme norm, we disclose the dynamics impact in ILC, which can be assessed by global uniform bound and monotonicity in iteration domain. Finally we address a rather controversial issue in ILC: can the higher-order ILC outperform the lower-order ILC in terms of convergence speed and robustness? By applying the min–max design, which is robust and optimal, and conducting rigorous analysis, we reach the conclusion that the Q-factor of ILC sequences of lower-order ILC is lower than that of higher-order ILC in terms of the time-weighted norm.

Keywords: Robust optimal design; Convergence speed; Dynamic impact; Higher-order ILC scheme; Convergence speed comparison

1. Introduction

In the last two decades iterative learning control (ILC) has been extensively studied, achieves significant progress in both theory and application, and becomes one of the most active fields in intelligent control and system control (Arimoto, 1985; Kawamura, Miyazaki, & Arimoto, 1987a; Hara, Yamamoto, Omata, & Nakano, 1988; Bien & Huh, 1989; Sugie & Ono, 1991; Kuc, Lee, & Nam, 1992; Jang, Choi, & Ahn, 1995; Saab, 1995; Amann, Owens, & Rogers, 1996; Phan & Juang, 1996; Lucibello, Panzier, & Ulivi, 1997; Lee & Bien, 1997; Longman & Lo, 1997; Moore, 1998; Chien, 2000; Lee & Lee, 2000; de Roover, Bosgra, & Steinbuch, 2000; Wang, 2000; Norrlof & Gunnarsson, 2001; Ham, Qu, & Kaloust, 2001; Xu & Tan, 2002). On the other hand, there are still numerous open problems left to researchers for further exploitation. In this paper, we address four open and most important issues in the field of ILC theory and design:

1. Can we design iterative learning controllers possessing robustness and optimality concurrently, in particular achieving fastest convergence in the presence of system uncertainties?
2. Can we evaluate learning convergence speed for various ILC schemes in a rigorous and quantitative manner?
3. How does the system dynamics affect the learning performance in iteration domain?
4. Can a higher-order ILC scheme perform better than lower-order ILC schemes?

It has been made clear that the ILC convergence is solely related to the system direct feed-through term. When the feed-through term is associated with a interval uncertainty, i.e. with lower and upper bounds, the idea of traditional ILC

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by the Associate Editor Tong Heng Lee under the direction of Editor Robert R. Bitmead

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PII: S0005-1098(02)00143-7

design is to guarantee the convergence condition for the worst case (upper bound), hence leads to a slower convergence speed due to the conservative design. Targeting at the time-optimal property and meanwhile assuring the learning convergence, we formulate the ILC design into a min–max optimization problem and provide a systematic solution. The max operation maximizes the influence from the system uncertainty, and min operation minimizes a learning factor that determines the convergence speed.

When a highly nonlinear, uncertain and non-affine system is under iterative learning control, it is very hard to guarantee that the performance of one ILC scheme is better than that of another one "uniformly" for all iterations. What is possible and more practical is to look for such indices that can capture the essential nature of an ILC scheme for "most" iterations. More rigorously speaking, one ILC scheme is thought of performing better than another in certain aspect, if the corresponding index of the former is better than that of the latter for infinitely many iterations except for a finite number of iterations. For this purpose, a learning performance index—Q-factor—is introduced, which provides a rigorous and quantified evaluation criterion for comparing the convergence speed of sequences generated by different ILC schemes. A lower Q-factor means a faster convergence speed for most iterations. Using Q-factor, it is easy to derive a "characteristic equation" that specifies the convergence speed for an iterative learning process.

In most ILC design and analysis, the system dynamic effect is neglected while a time weighted norm is used. In this paper, we investigate the relationship between the system dynamic influence and the time-weighted norm. In order to quantify the dynamic impact to the learning process, we introduce two indices with supreme norm—the global uniform bound of tracking error in iteration domain, and monotonicity period. The former describes the worst case error bound, and the latter specifies the maximum tracking interval in which the tracking error decreases monotonically in terms of the supreme norm. By means of these two indices, the system dynamic impact, which is hidden and suppressed by the time-weighted norm, is clearly exhibited.

The last issue is rather controversial in ILC. Intuitively, a higher-order ILC, that employs preceding control information of more than one iteration, should be able to improve learning performance as more of preceding control information is used. However, a simple linear combination of preceding control information may not provide new information. Note that most higher-order ILC schemes proposed hitherto are of linear type. What is more, for a convergent ILC sequence, in most iterations the latest should be the most accurate and the rest are less. A linear combination of less accurate ones may further degrade the performance. To answer this question, rigorous analysis and fair comparisons are indispensable. In the last part of this paper we analyze and compare the learning convergence speed associated with linear first-order and higher-order ILC schemes. Based on the min–max design and Q-factor, we are able to conduct a quantitative comparison and reach the following conclusion. Under the same interval uncertainty and applying the same min–max design which is robust and optimal, the Q-factor of ILC sequences of lower-order ILC is lower than that of higher-order ILC in terms of the time-weighted norm. In the sequel, the first-order ILC achieves the fastest convergence speed in the iteration domain in the sense of Q-factor.

This paper is organized as follows. The learning control problem is formulated in Section 2. Section 3 presents the convergence analysis and robust optimal design for the first-order ILC scheme under interval uncertainty. Section 4 explores the dynamic impact in iteration domain. Section 5 compares the learning convergence speed for higher-order ILC schemes.

2. Problem formulation

Consider the nonlinear dynamic system (1),

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), u(t), t) \quad \mathbf{x}(0) = \mathbf{x}_0, \\ y(t) &= g(\mathbf{x}(t), u(t), t), \end{aligned} \tag{1}$$

where $t \in [0, T]$, $\mathbf{x}(t) \in R^n$, $y(t) \in R$ and $u \in R$, $\mathbf{f}(\cdot)$ and $g(\cdot)$ are partially unknown functions. The system is satisfying the following assumptions.

Assumption 1. Denoting $\Omega \triangleq R^n \times R \times [0, T]$, $0 < \alpha_1 \leqslant \partial g/\partial u \leqslant \alpha_2$ and $\|\partial g/\partial \mathbf{x}\| \leqslant \beta_{\mathbf{x}}$, $\forall (\mathbf{x}, u, t) \in \Omega$. Here α_1, α_2 are known constants and $\beta_{\mathbf{x}}$ is an unknown constant.

Remark 1. $\partial g/\partial u$ is equivalent to system direct feed-through term and represents the system gain. $0 < \alpha_1 \leqslant \partial g/\partial u$ warrants no singularity in the system control. $\partial g/\partial u \in D \triangleq [\alpha_1, \alpha_2]$ indicates the presence of an interval uncertainty in the system gain, which directly affects the control performance. One of the objectives of this paper is to develop an appropriate learning control design so as to achieve both robustness and optimality in the presence of the interval uncertainty D.

Assumption 2. Nonlinear function $\mathbf{f}(\mathbf{x}, u, t)$ is global Lipschitz continuous with respect to $\mathbf{x}$ and u in the set Ω, i.e.,

$$\|\mathbf{f}(\mathbf{x}_1, u_1, t) - \mathbf{f}(\mathbf{x}_2, u_2, t)\| \leqslant L_f[\, \|\mathbf{x}_1 - \mathbf{x}_2\| + |u_1 - u_2|],$$

where L_f is an unknown Lipschitz constant.

Assumption 3. For the given trajectory $y_r(t)$, there exists a unique $u_r(t)$ such that

$$\dot{\mathbf{x}}_r(t) = \mathbf{f}(\mathbf{x}_r(t), u_r(t), t)$$

$$y_r(t) = g(\mathbf{x}_r(t), u_r(t), t) \quad \forall t \in [0,\ T].$$

Remark 2. Since $u_r(t)$ exists uniquely, the uniform convergence of the control profile $u(t)$ to $u_r(t)$ implies that the state and output tracking errors will vanish.