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Reward–risk portfolio selection and stochastic dominance

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Abstract

The portfolio selection problem is traditionally modelled by two different approaches. The first one is based on an axiomatic model of risk-averse preferences, where decision makers are assumed to possess a utility function and the portfolio choice consists in maximizing the expected utility over the set of feasible portfolios. The second approach, first proposed by Markowitz is very intuitive and reduces the portfolio choice to a set of two criteria, reward and risk, with possible tradeoff analysis. Usually the reward–risk model is not consistent with the first approach, even when the decision is independent from the specific form of the risk-averse expected utility function, i.e. when one investment dominates another one by second-order stochastic dominance. In this paper we generalize the reward–risk model for portfolio selection. We define reward measures and risk measures by giving a set of properties these measures should satisfy. One of these properties will be the consistency with second-order stochastic dominance, to obtain a link with the expected utility portfolio selection. We characterize reward and risk measures and we discuss the implication for portfolio selection.

JEL classification: G11; D81
Keywords: Stochastic dominance; Coherent risk measure; Decision under risk; Mean-risk models; Portfolio optimization

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1. Introduction

Markowitz (1952) introduced an intuitive model of return and risk for portfolio selection. This model is useful to guide one's intuition, and because of its simplicity it is also commonly used in practical finance decisions. Markowitz (1952) proposed to model return and risk in term of mean and variance, but he also suggested other measures of risk as for example the semivariance (Markowitz, 1959).

The advantage of using the variance for describing the risk component of a portfolio, is principally due to the simplicity of the computation, but from the point of view of risk measurement the variance is not a satisfactory measure. First, the variance is a symmetric measure and "penalizes" gains and losses in the same way. Second, the variance is inappropriate to describe the risk of low probability events, as for example the default risk. Third, the mean–variance approach is not consistent with second-order stochastic dominance and thus with the expected utility approach for portfolio selection. This is well illustrated by the (μ, σ)-Paradox (see Copeland and Weston, 1998, Chapter 4.G).

As already suggested by Markowitz (1959), Ogryczak and Ruszczynski (1997) also proposed semivariance models, where the reward–risk approach is maintained, but the choice of semivariance instead of variance makes the model consistent with second-order stochastic dominance. They also extend the consistency concept to higher order stochastic dominance by defining a more general central semideviation measure. Other risk measures have been proposed for portfolio selection, as for example Value-at-Risk (Jorion, 1997, Duffie and Pan, 1997) or Expected-Shortfall (Acerbi and Tasche, 2002, Bertsimas et al., 2004). The latter one is consistent with second-order stochastic dominance, as illustrated by Bertsimas et al. (2004), who introduced Expected-Shortfall exactly because its consistency with second-order stochastic dominance. Value-at-risk is widely used in practice, but it is only consistent with respect to first-order stochastic dominance (see Hürlimann, 2002a). Moreover, it has been shown by Artzner et al. (1997), that value-at-risk fails in controlling the risk of large losses with small probability, since it only considers the probability of certain losses to occur, but not the magnitude of these losses. Moreover, value-at-risk usually does not satisfy the subadditivity property, which ensures – if satisfied – a reasonable behavior of the risk measure when adding two positions. Artzner et al. (1999), being concerned with banking regulations, have proposed an axiomatic approach to the definition of a risk measure. They presented a set of four properties for measures of risk and they called measures satisfying these properties, *coherent risk measures*. Moreover, they show that a coherent risk measure can be still characterized by a non-empty set of scenarios (called generalized scenario), such that "any coherent risk measure arises as the supremum of the expected negative of final net worth for some collection of probability measures on the states of the world" (Artzner et al., 1999, Section 4.1). Unfortunately, coherent risk measures are usually not consistent with second-order stochastic dominance. Pflug (1998) considered various classes of risk measures and gave the general prop-