Available online at www.sciencedirect.com
ScienceDirect
Automatica 42 (2006) 2079–2089
automatica
www.elsevier.com/locate/automatica

On admissible pairs and equivalent feedback—Youla parameterization in iterative learning control[☆]

Mark Verwoerd[a,*], Gjerrit Meinsma[b], Theo de Vries[b]

[a]Hamilton Institute, National University of Ireland, Maynooth, Co. Kildare, Ireland
[b]Department of Electrical Engineering, Mathematics, and Computer Science, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

Received 19 December 2003; received in revised form 26 May 2006; accepted 29 June 2006
Available online 1 September 2006

Abstract

This paper revisits a well-known synthesis problem in iterative learning control, where the objective is to optimize a performance criterion over a class of causal iterations. The approach taken here adopts an infinite-time setting and looks at limit behavior.

The first part of the paper considers iterations without current-cycle-feedback (CCF) term. A notion of admissibility is introduced to distinguish between pairs of operators that define a robustly converging iteration and pairs that do not. The set of admissible pairs is partitioned into disjoint equivalence classes. Different members of an equivalence class are shown to correspond to different realizations of a (stabilizing) feedback controller. Conversely, every stabilizing controller is shown to allow for a (non-unique) factorization in terms of admissible pairs. Class representatives are introduced to remove redundancy. The smaller set of representative pairs is shown to have a trivial parameterization that coincides with the Youla parameterization of all stabilizing controllers (stable plant case).

The second part of the paper considers the general family of CCF-iterations. Results derived in the non-CCF case carry over, with the exception that the set of equivalent controllers now forms but a subset of all stabilizing controllers. Necessary and sufficient conditions for full generalization are given.

Keywords: Iterative learning control; Youla parameterization; Feedback control; H_∞ control; Equivalent control; Linear systems

1. Introduction

Some 20 years ago, Arimoto, Kawamura, and Miyazaki (1984) were among the first to develop a theory of learning specifically tailored to single-loop control problems. Upon observing the human tendency to learn from experience, they were led to ask whether it would be possible to implement a similar ability in the automatic operation of dynamical systems. In answer, they proposed a ‘betterment process’, now known as *iterative learning control* (ILC). The method proved effective and inspired a great number of researchers. Over the years, Arimoto’s original algorithm has been modified and extended in a number of ways: assumptions have been relaxed, robustness has been improved, and convergence properties have been laid out in detail. See Moore, Dahleh, and Bhattacharyya (1992) and Amann, Owens, and Rogers (1994) for an overview of early results. Today, there is an extensive literature covering a wealth of different learning rules applicable to a wide range of systems, both linear and nonlinear. Recent surveys include Moore (1999), Chen and Wen (1999), and Xu and Tan (2003).

In the early days, questions of analysis and synthesis were addressed almost exclusively within a time-domain framework which was built around the finite-trial-length postulate (Arimoto, 1998, chap. 1; Arimoto et al., 1984). This framework became the standard for many years and is still among the most commonly used today. Yet, over the course of two decades, a variety of other techniques have been introduced, some to considerable effect. This paper is about one such technique. The technique in question originates in the early nineties, when, following developments in the general field of control, people

[☆] This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Gang Tao under the direction of Editor Miroslav Krstic.

* Corresponding author.
E-mail addresses: mark.verwoerd@nuim.ie (M. Verwoerd), g.meinsma@utwente.nl (G. Meinsma), t.j.a.devries@utwente.nl (T. de Vries).

0005-1098/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2006.06.022

begin to view ILC as an H_∞ synthesis problem. The synthesis problem is to minimize a performance criterion, typically the mean-squared tracking error, over the space of bounded (real-rational) transfer functions, RH_∞. See, for example, Padieu and Su (1990), Kavli (1992), Amann, Owens, Rogers, and Wahl (1996), and Moore et al. (1992). In this approach the finite-trial-length postulate is dropped and an extra assumption introduced, namely that learning operators be *causal* (recall that every element in RH_∞ defines a *causal* bounded (finite-dimensional) LTI operator). At the time, few would have anticipated that as natural a role as causality plays in conventional feedback control, as restrictive and unnecessary it would prove in the context of ILC. The success of Arimoto's learning rule was known to be tied up with the availability of a *future* error (i.e. with a *non*-causal ingredient of some kind). And indeed, the fact that a learning operator need not be causal was well-established (Moore et al., 1992; Moore, 1999). Yet it would seem that the precise reason as to why causality should affect ILC performance the ways it does was not known. In contrast, the implications for compensator design were well-understood, owing largely to the classical work of Bode. By the time Goldsmith (2001, 2002) introduced the notion of equivalent feedback, the fact that causality seriously impairs the achievable performance was widely acknowledged, and studies into the merits of non-causal ILC were well-underway. Indeed, the thesis that a non-causal approach would constitute 'the only viable route for ILC' (Goldsmith, 2002, p. 708) had been voiced by others before. It would appear however that Goldsmith was the first to provide compelling evidence for it. The evidence has been contested (see Owens & Rogers, 2004; Goldsmith, 2004) but as of yet the thesis has not been overthrown. The work presented in this paper builds on that of Goldsmith's. We provideseveral extensions, most notably a converse result, which states that the set of equivalent controllers is generally but a subset of all stabilizing controllers. Conditions under which both sets coincide are given. Also, we state precise conditions (as captured by our notion of admissibility) under which causal ILC and conventional feedback are equivalent and provide an example of a causal iteration with an equivalent controller that is destabilizing.

Following Padieu and Su (1990), Kavli (1992), Moore et al. (1992), Moore (1993), de Roover (1996) and Amann et al. (1996), among others, this paper poses the problem of ILC as a two-parameter synthesis problem. The parameters are assumed causal bounded operators acting on the current input and current error, respectively. Our approach comprises the following steps. First, a notion of admissibility is introduced. This notion is used to single out 'bad' pairs of operators. Then the two-parameter problem is shown to be overparameterized; that is, different admissible pairs are shown to induce different sequences converging to the same fixed point. Redundancy is removed by grouping 'equivalent pairs' into equivalence classes, restricting attention to class representatives. Finally, the resulting one-parameter problem is shown to be a standard compensator design problem. The organization of the paper is as follows. Section 2 introduces the problem of ILC. Two problem cases are identified and discussed in subsequent sections: standard ILC in Section 3 and current-cycle-feedback-ILC (CCF-ILC) in Section 4. Section 5 closes with conclusions and recommendations.

2. Iterative learning control

2.1. Problem statement

Let U, Y be vector spaces. Given a plant $P : U \mapsto Y$, along with some $y_\mathrm{d} \in Y$, the problem of ILC is to construct a sequence $\{u_i\}$, with $u_i \in U$ for all i, such that $\{(u_i, Pu_i)\}$ converges to a limit point in $U \times Y$ and $\bar{y} := \lim_{i\to\infty} Pu_i$ is close to y_d. We shall assume that P is LTI and finite dimensional, i.e. that $P(s)$ is real-rational. Throughout the paper, the space of inputs U is $L_2[0, \infty)$ (H_2).

Let us consider an example. Let $P : L_2[0, \infty) \mapsto L_2[0, \infty)$ be defined as

$$(Pu)(t) := \int_0^\infty \mathrm{e}^{-(t-\tau)} u(\tau)\,\mathrm{d}\tau, \quad t \geqslant 0 \tag{1}$$

and let y_d be given as $y_\mathrm{d} := Pu_\mathrm{d}$, where

$$u_\mathrm{d} := \begin{cases} t(1-t) & \text{for } 0 \leqslant t < 1 \\ 0 & \text{for } t \geqslant 1. \end{cases} \tag{2}$$

We select $u_0(t) := \mathrm{e}^{-t}\sin(t)$, $t \geqslant 0$, and recursively define

$$u_{k+1}(t) := u_k(t) + e_k(t), \quad k = 0, 1, \ldots . \tag{3}$$

Here, $e_k(t) := y_\mathrm{d} - Pu_k(t)$ denotes the current tracking error. Focusing on the system's output on the finite interval $[0, 5]$ we are interested in the evolution of the mean-squared error (MSE), $\|e_k\| := \frac{1}{5}\int_0^5 \mathrm{e}_k^2(t)\,\mathrm{d}t$, as a function of the iteration number, k. Fig. 1 shows that the MSE progressively decreases, and does so in an exponential fashion. Also shown is the system's output after 10 trials, $y_{10}(t)$, along with the desired output $y_\mathrm{d}(t)$. This concludes the example.

We remark that system (3) may equivalently be defined as

$$u_{k+1}(s) := u_k(s) + e_k(s), \quad k = 0, 1, \ldots, \tag{4}$$

where $u_k(s)$ and $e_k(s)$ denote the images of $u_k(t)$ and $e_k(t)$ under the one-sided Laplace transform. In the sequel we shall adopt this so-called frequency-domain representation. Note that this is a matter of preference and does not encompass any loss of generality. In the remainder of the paper it is assumed that the sequence $\{u_k\}$ is generated by an element of the iteration class $T_C(Q, L) : H_2 \to H_2$,

$$u_{k+1} := Qu_k + Le_k + Ce_{k+1}, \quad k = 0, 1, \ldots \tag{5}$$

and that Q and L take values in RH_∞. Here $e_k := y_\mathrm{d} - y_k$ is the (current) tracking error we introduced earlier. The parameter C represents a fixed feedback map and is not considered a design parameter. Lastly, the interconnection of P and C is assumed