Simulation Modelling Practice and Theory 44 (2014) 54–62

Contents lists available at ScienceDirect

Simulation Modelling Practice and Theory

journal homepage: www.elsevier.com/locate/simpat

Application of global sensitivity analysis to a tire model with correlated inputs

Floriane Anstett-Collin [a,*], Thierry Mara [b], Michel Basset [c]

[a] Université de Lorraine, 2 Rue Jean Lamour, 54519 Vandoeure Lès Nancy Cedex, France
[b] Université de La Réunion, 15 Avenue René Cassin, 97715 Saint-Denis Cedex 9, La Réunion, France
[c] Université de Haute Alsace, 12 Rue des frères Lumière, 68093 Mulhouse Cedex, France

ARTICLE INFO

Article history:
Received 26 September 2012
Received in revised form 28 February 2014
Accepted 12 March 2014
Available online 13 April 2014

Keywords:
Global sensitivity analysis
Correlated parameters
Polynomial chaos expansion
Tire model

ABSTRACT

When a vehicle equipped with tire is manoeuvred on the ground, the tires are submitted to a number of forces – longitudinal force when driving or braking torque is applied to the wheel and/or lateral force when the wheel is steered to turn at a corner. Pacejka model describes these forces that represent the reaction of the road onto the tire. This nonlinear model depends on correlated parameters such as the friction coefficient, the vertical load, and the cornering stiffness, which have to be identified from some measurements. The sensitivity of Pacejka model to these correlated parameters are studied using an approach based on polynomial chaos. It consists in decorrelating the parameters using the Nataf transformation and then, in expanding the model output onto polynomial chaos. The sensitivity indices are then obtained straightforwardly from the algebraic expression of the coefficients of the polynomial expansion.

1. Introduction

In the automotive and aeronautical fields, modelling the tire/road interface is fundamental. Indeed, the tire model is one of the major elements to integrate into a ground vehicle or aircraft model, as the tires are the only contact surface with the road. When a vehicle equipped with tires is manoeuvred on the ground, they are subject to a number of forces – longitudinal force when driving or braking torque is applied to the wheel and lateral force when the wheel is steered to turn at a corner. The tire models available in the literature describe the efforts and moments corresponding to the road reaction of the tire (see for instance [1–4]).

One of the most famous model was proposed by Pacejka [1] and is often used [5] nowadays by industrials (Michelin, SAE, Adams tire software, etc.). This model is nonlinear, complex and depends on a certain number of parameters (friction coefficient, vertical load, side slip angle, etc.) that can be obtained from experimentation. Unfortunately, experimental data are often sparse or incomplete, especially in the aircraft domain and their measurements are very expensive. On the one hand, some parameters only have a negligible influence on the model response and therefore, do not need to be determined accurately. On the other hand, some others are relevant for the model response and thus influence its uncertainty significantly. These parameters may require additional measurement effort in order to be estimated with relatively high accuracy. In order to prepare and plan future experiments, it is necessary to perform a sensitivity analysis of the Pacejka model.

* Corresponding author. Tel.: +33 (0)3 83 685 139.
E-mail address: floriane.collin@univ-lorraine.fr (F. Anstett-Collin).

http://dx.doi.org/10.1016/j.simpat.2014.03.003
1569-190X/

Several studies have focused on the global sensitivity analysis of nonlinear models with independent inputs (for instance [6–12]). They rely on variance-based sensitivity indices also known as Sobol' indices [7]. They measure the contribution of input parameters to the model response variance. Such a contribution can be due to a parameter alone or to a group of parameters. Sobol' indices are unique and easily interpretable when the parameters are independent. But, it is more challenging when they are dependent. Indeed, if a function (*i.e.* model response) structurally depends on parameter p_1 and not on p_2, and if p_1 and p_2 are correlated, classical Sobol' indices will lead to the conclusion that both are relevant inputs, even though actually p_2 is influential only because of its correlation with p_1.

It may be of high interest to distinguish whether an input is relevant regarding its correlations with the other ones [13]. This information may be of great help for the experimenter in order to guide future experiments. Mara and Tarantola [14] have derived a set of variance-based sensitivity indices to cope with parameters dependency. They also proposed a computational method for their estimation. The proposed approach relies on the use of polynomial chaos expansion [15] in conjunction with a Gram–Schmidt based decorrelation procedure. They also indicate that Nataf [16] or Rosenblatt [17] transformations can be used.

This study is part of a french national project involving several industrial partners (Airbus, Messier-Dowty, Turbomeca, etc.) and research centers, in particular the Modélisation Intelligence Processus Systèmes (MIPS) laboratory. The aim of the project is to take advantage of performances of the modern simulation tools by providing new models and developing tools which allow the simulation of systems and physical phenomenon in the aeronautical fields. More precisely, the laboratory works to develop models for simulating the tyre–road interaction characteristics with respect to the aircraft run types. In this framework, the aim of the paper is to analyze the well-known Pacejka tire model, which is a basic function of the Magic Formula [1]. Pacejka model is widely used in the automotive and aeronautical fields. The model depends on correlated parameters (friction coefficient, cornering stiffness, vertical load, etc.) which must be identified from measurement data. The most relevant parameters on the lateral force are highlighted, using the approach proposed in [14].

The paper is organized as follows. Section 2 presents the Pacejka tire model. Section 3 recalls the expression of the Sobol' sensitivity indices for model with independent parameters and their estimation based on polynomial chaos. Section 4 presents the approach used to study the sensitivity for models with correlated parameters. Then, the sensitivity analysis of the Pacejka tire model is performed.

2. Pacejka tire model

Tires in motion on the ground are subject to a number of forces. For example, a longitudinal force is developed when driving or braking torque is applied to the wheel. A lateral force appears when the wheel is at an angle or when it is steered to turn at a corner. The model considered in this study accounts for the longitudinal force (driving or braking force) and the lateral force. In the present study, we will exclusively focus on the model expression for the lateral force developed in the case of a cornering manoeuvre in steady-state condition, as represented in Fig. 1.

In pure cornering condition, an interpolation function, called Magic Formula [1], is proposed for the lateral force. The Pacejka model presented here is a basic function of the Magic Formula. In this case, the lateral force is given by:

$$F_y = D\sin[C\arctan(B(\alpha+S_h) - E(B(\alpha+S_h) - \arctan(B(\alpha+S_h))))] + S_V \quad (1)$$

The model output of interest F_y is the lateral force, road reaction in the lateral direction. The parameter α is the side slip angle, that is the angle between the wheel plane and the wheel direction of motion (see Fig. 1). The parameter D represents the maximum value that F_y can reach and C, E, S_h and S_v are empirical fitting parameters. Fig. 2 depicts the relationship

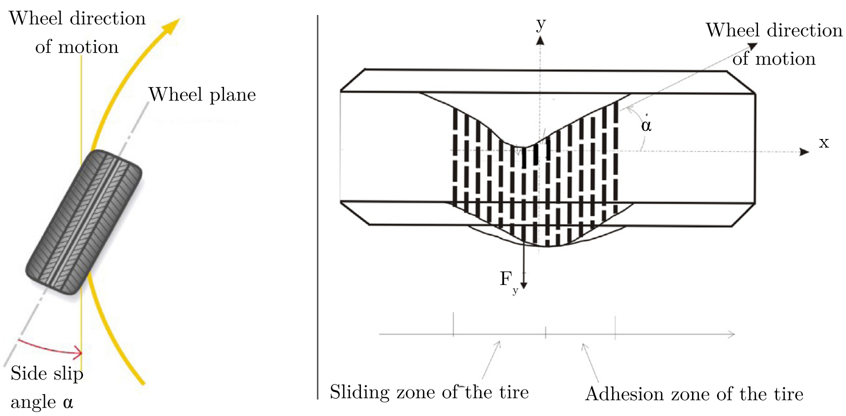

Fig. 1. Pure cornering – General view of the tire deformation in the contact patch.