



Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: [www.elsevier.com/locate/ejmflu](http://www.elsevier.com/locate/ejmflu)

## Sensitivity analysis of optimal transient growth for turbulent boundary layers

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### ARTICLE INFO

#### Article history:

Available online xxxx

#### Keywords:

Turbulent boundary layer  
 Optimal transient growth  
 Sensitivity to mean flow modification

### ABSTRACT

Structural approaches based on modal decomposition of the flow dynamics have gained acceptance for a wide variety of turbulent shear flows. In this context, a singular value decomposition associated with a governing operator, aiming to model the linear amplification of coherent structures, is used to reproduce some fundamental motions in a turbulent boundary layer. In particular, as already found by Cossu et al. (2009), elongated streaky structures scaled in inner and outer units are identified. The sensitivity of these singular values to a mean flow modification is analysed. It is illustrated that the linear amplification of very large-scales which populate the outer motion is not affected when the leading singular value associated with the inner layer is damped. Moreover, we notice that the resulting optimal mean flow deviation is consistent with findings of Xu et al. (2007) in which the active control of a turbulent boundary layer is studied through direct numerical simulations.

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### 1. Introduction

Since the pioneer work of Kline et al. [1] devoted to the understanding of the role of coherent structures in turbulent boundary layers (as streaks, vortices and hairpins), the structural point of view of wall turbulence gives new insight about turbulence dynamics (see the recent review of Jiménez [2]). The study of coherent structures, their spatial and time scales, their mutual interactions as well as their relationships, is fundamental to develop low-order models allowing us to describe mechanisms involved in wall turbulence (see Panton [3]). In that respect, as first suggested by Hamilton et al. [4], it is now commonly accepted that a self-sustaining process near the wall plays a major role in both the large-scale dynamics and the production of turbulent kinetic energy. The latter process relies on the formation of streamwise velocity streaks and their nonlinear breakdown. The linear part of such a process is caused by a so-called lift-up mechanism. Jiménez [5] also stressed that, in spite of the turbulence requires nonlinearity, there is strong evidence that the emergence of coherent structures as well as the main part of energy production mechanism in wall-bounded turbulence can be described by a linear model.

Structural approaches based on a linearization about a turbulent mean flow were first introduced by Reynolds and Hussain [6]. A linear dynamical system for the organized motion is specified where a triple decomposition of the instantaneous flow fields, which decorrelates the random part from the coherent part of the motion, is used. To model the coherent part of the Reynolds stress, the authors introduce an eddy viscosity by assuming that the large-scales feel the dissipation from the smaller ones. However, by considering only asymptotic solutions, their analysis fails to describe accurately the coherent structures of a turbulent channel flow. In 2006, Del Alamo and Jiménez [7] reconsidered the linear model, proposed by Reynolds and Hussain [6], by tracking coherent structures which may be sustained in short times. The most amplified modes are computed for given time horizons and then compared with the turbulent structures in a channel flow. The analysis shows that the coherent structures which gain the maximum energy in the transient time reproduce the organization of turbulence in a channel flow both in terms of spatial and temporal scales. Such an attempt extends the previous results obtained by Butler and Farrell [8] in 1992 who used a similar transient growth theory (see Schmid and Henningson for a review in a laminar regime [9]) where the influence of the background turbulence onto the large-scale motion is not included in the equations. To take into account the effect of the dissipation due to the turbulence, Butler and Farrell [8] constrain the time horizon with an appropriate turbulent eddy turnover time. Further investigations carried out by

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Pujals et al. [10] for a turbulent channel flow and Cossu et al. [11] for a turbulent boundary layer confirm the relevancy of linear optimal transient growth analyses, capable of providing characteristic features of wall-bounded flows. In particular, the previous authors emphasize that the modes are mostly amplified for infinitely elongated structures in the streamwise direction. In addition, the curve associated with the maximum energy gain exhibits two optima. For the turbulent boundary layer, Cossu et al. [11] show that an optimum in energy gain is scaled in inner units with a spanwise wavelength  $\lambda_z^+ \approx 81$  ( $\lambda_z^+ = \lambda_z / \delta_w$  with  $\delta_w$  the wall viscous length scale). This spatial scale is consistent with the size of streaky structures involved in the autonomous cycle of near-wall turbulence [3]. A second optimum is scaled in outer units and is also seen to widely extend in the inner region. This observation is in agreement with experiments which report a scaling in outer units for the very large-scale motion lying in the outer region and a footprint of these structures in the inner layer [12]. Nevertheless, both the streamwise and spanwise wavelengths are overestimated by the most amplified structure whereas suboptimal modes are more consistent with experimental observations. More recently, Sharma and McKeon [13] described additional features in a turbulent channel flow by computing optimal response modes, i.e. modes leading to an optimal gain in energy with respect to a localized forcing. For instance, an appropriate superposition of the leading optimal modes predicts the organization of hairpin packets and the dynamics of very large-scale motion in the outer region, as described by Kim and Adrian [14]. Sharma and McKeon [13] illustrate that such very large-scale structures may arise naturally, rather than by a summation of hairpin packets. This theoretical analysis contradicts thus the model proposed by Kim and Adrian [14].

The singular value decomposition of a linear operator, the so-called propagator in the time domain and the resolvent in the frequency domain, is a common aspect of all the previous studies referenced above. Hence, the optimal modes associated with the most amplified singular values appear relevant to describe some fundamental features of coherent motions in turbulent shear flows. Nevertheless, such a linear analysis fails until now to predict some mutual dependency between the large-scale structures associated with the outer region and the coherent structures which populate the inner region. More particularly, as underlined by Mizuno and Jimenez [15], the mutual interactions between the inner and outer layers for wall-bounded turbulence remain an open question. For example, how the large-scale motion in the outer region would continue to exist in the absence of the near-wall structures. Hwang and Cossu [16] have recently tackled this fundamental question through a Large Eddy Simulation of a turbulent channel flow with a Smagorinsky subgrid model. The authors show that an increase of the Smagorinsky constant is able to damp the small-scale coherent motion near the wall. As a consequence, the near-wall self-sustaining cycle is suppressed. The large-scale structures associated with the outer region are not modified. This analysis highlights that the large-scale dynamics in the outer region are almost independent of the near-wall cycle which dominates in the buffer layer. One may also remark that such an analysis further supports Townsend's wall similarity hypothesis [17] for rough walls stating that turbulence outside the inner layer is unaffected by surface condition. This hypothesis is quite controversial and is the scope of extensive research in both experiments [18–20] and numerical simulations [21–23]. Although Townsend's wall similarity hypothesis is still an open problem, there is some evidence that a part of discrepancies observed in the literature are caused by the different roughness geometries that are used [24].

From the above discussion, it seems relevant to assume that the linear optimal transient growth model used by Del Alamo and Jiménez [7], Pujals et al. [10] and Cossu et al. [11] is appropriate to describe both the inner layer and the outer layer coherent

structures. Hence, in order to support the scenario proposed by Hwang and Cossu [16], one may investigate the amplification of optimal modes associated with the outer region when the most amplified singular value for the inner region is damped. It is interesting to further explore this fundamental question by means of a theoretical analysis based on the sensitivity of the leading singular values to mean flow modifications. A mean flow modification may be caused, for instance, by riblets, roughness elements or an actuator. In a linear stability framework, Bottaro et al. [25] seek to determine the effect on the asymptotic linear stability of a base flow deviation, of a given amplitude, from the ideal base flow. By introducing a so-called sensitivity function, a standard variational procedure is employed to target the optimal base-flow modification that maximizes (or minimizes) the temporal growth rate of a selected eigenmode, when assuming a fixed deviation parameter. A transition scenario involving the exponential growth of small disturbances is thus proposed by Bottaro et al. [25] in order to explain the triggering of turbulence for asymptotically linearly stable parallel shear flows as Couette flow. This concept is extended by Marquet et al. [26] in the control of the onset of unsteadiness behind a cylinder in a laminar regime. More recently, Meliga et al. [27] push forward such theoretical tools to control large-scale unsteadiness in a turbulent wake past a D-shaped cylinder. The latter authors compute the natural shedding mode by a linear stability analysis performed on the linearized U-RANS equations. Hence, Meliga et al. [27] assess sensitivity maps for the shedding mode with respect to small turbulent mean flow deviation caused by a small control cylinder. In particular, they identified the regions in which the small control cylinder has the most impact on both the frequency and the amplification rate of the shedding mode. All analyses discussed above focus on asymptotic behaviour. Brandt et al. [28] extend such a concept by considering the sensitivity of the optimal transient energy growth to a base flow modification, i.e. the leading singular value of the governing operator. The flat plate laminar boundary layer is used to illustrate their purpose. After having derived an analytical expression for the sensitivity function, the authors show the effect of a small base flow deviation on the lift-up mechanism and also its impact on the Tollmien–Schlichting waves. It is thus clear that sensitivity to mean flow modification analyses allows us to identify a transition scenario, mutual interactions between two competitive mechanisms, as well as passive control strategies.

The present research is an attempt to push forward the theoretical concepts of Brandt et al. [28] for turbulent flows. For that purpose, the sensitivity framework detailed in [28] is extended to the governing operator used to compute coherent structures in turbulent shear flows by [7,10,11]. More particularly, we would like to emphasize the mutual interaction between the inner and outer layer large-scale motions and also to provide a theoretical framework to damp the autonomous cycle of near-wall turbulence. To achieve such a goal, the line of thought is as follows: after having presented the turbulent boundary layer mean flow profiles, we address the basic concepts of optimal transient growth and sensitivity analysis for a turbulent mean flow. Then, we present the sensitivity functions for a mean flow modification for both the optimal modes associated with the inner and outer regions. The optimal mean flow deviation which stabilizes the optimal mode for the inner region is thus tackled. Hence, the mutual interactions between the inner and outer large-scale motions are highlighted, as well as the optimal distorted turbulent mean flow. A final part is dedicated to some discussions and prospects.

## 2. Mean flow and eddy viscosity

The explicit expression for the turbulent mean velocity given by Monkewitz et al. [29] is used

$$U^+ = U_i^+(y^+) - U_{\log}^+(y^+) + U_e^+(Re_{\delta^*}) - U_w^+(\eta),$$

$$\text{with } U^+ = U/u_\tau \quad (1)$$

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