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## Sensitivity analysis of the non-destructive evaluation of micro-cracks using GMR sensors

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### ABSTRACT

Micro-cracks in a magnetized ferromagnetic material cause stray fields that can be observed using giant magnetoresistive (GMR) sensors. This work investigates the applicability of GMR sensors to the non-destructive evaluation of micro-cracks via the observation of stray fields. For this purpose, our measurement setup is assessed using a fast new sensitivity analysis based on adjoint states, employing the finite-element method. A model for the GMR sensor is developed and verified. We are able to resolve micro-cracks with an opening of 3  $\mu\text{m}$  and a depth of 30  $\mu\text{m}$ . GMR positioning inaccuracies are analyzed.

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### 1. Introduction

Magnetic stray fields are caused by a change in the distribution of the magnetic properties of a magnetized ferromagnetic object. Their observation in magnetic flux leakage investigations can allow the non-destructive detection and evaluation of material defects [1]. The assessment of micro-cracks can be important in the safety testing of ferromagnetic components. Quantitative magnetic field measurements can be achieved using the giant magnetoresistive (GMR) effect to produce small elements highly sensitive to magnetic fields [2]. GMR sensors measure a projection of the magnetic vector field in one spatial direction. Despite their small size, GMR sensing areas cannot be considered point-like with respect to micro-cracks [3]. Therefore, a realistic sensor model is required for simulations. Inaccuracies in the measurement setup often degrade the quality of detection and the results of the non-destructive evaluation.

The aim of this study is to characterize our experimental setup for measurements of magnetic flux leakage using GMR sensors. Sensitivity analysis can provide information about a specific setup's detection and identification abilities and indicate possible improvements [4]. It provides gradient information necessary for optimization procedures used for solving inverse problems. For our future evaluation, we are interested in computing the

magnetic properties that produce the measurement data. The underlying inverse problem is under-determined; it follows that the number of unknown magnetic parameters greatly exceeds the number of measurements. For the magneto-static case, the problem can be formulated with regard to a few parameters that define the defect geometry. Then an analytical solution of the corresponding sensitivity equations can be computed quickly and accurately [5]. Magnetic flux leakage analysis with respect to the experimental design can be found in [6–8]. In [7], a sensitivity analysis for sensor lift-off variations based on analytic stray field equations was performed. For the electromagnetic case, the reciprocity theorem allows quick and accurate sensitivity analysis with only two solutions of the forward problem [9,10]. Probabilistic uncertainty and sensitivity analysis is a popular area of research, see for instance [11]. In this paper, we reformulate deterministic sensitivities in terms of adjoint states, as introduced in [12]. We focus particularly on the sensitivity analysis of resolving micro-cracks using GMR measurements. The sensitivity analysis involves a finite-element forward model in order to represent material damages in terms of changes in the distribution of the magnetic permeability. An advantage of the adjoint state formulation is the small number of required finite-element approximations: one simulation for the forward problem and one simulation for each corresponding measurement are required for each updated magnetic material parameter distribution. Therefore, quicker computation is achieved in comparison with finite-difference quotient analysis. The reformulation of sensitivities is achieved analytically, and we expect to reduce the overall numerical approximation error.

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We apply the new adjoint states sensitivity method to the analysis of magnetic flux leakage measurement using GMR sensors. We investigate the detection limits for micro-cracks with respect to measurement uncertainties such as sensor position and orientation. The properties of the linearized inverse problem, which describe the reconstruction of the micro-cracks, are assessed in terms of model resolution analysis and condition numbers.

## 2. Methods

### 2.1. Forward problem

The forward problem in terms of calculating magnetic stray fields due to a change in the magnetic permeability can be described by the magneto-static form of Maxwell's equations, which can be solved numerically by finite-element analysis. Then, Eq. (1) describes the nonlinear mapping of the source parameters  $\mathbf{x}$ , i.e., via the operator  $\mathbf{F}$  onto the signal observed by the GMR sensor

$$\mathbf{y} = F(\mathbf{x}). \quad (1)$$

The sensitivity matrix  $\mathbf{S} = \partial \mathbf{y} / \partial \mathbf{x}$  describes how changes of the magnetic permeability distribution lead to changes of the sensor signals. It contains information about the parametrization of the source region and the behavior of the sensor (or the distribution of sensors) in the observation region.

The magnetic permeability is a material property. It describes the input for each finite-element analysis and is the variable for which we perform the sensitivity analysis. Ferromagnetic materials require in general to consider a second rank nonlinear permeability tensor

$$\mathbf{x}_i = \mu_i = \begin{pmatrix} \mu_{xx}(H_{xx}) & \mu_{xy}(H_{xy}) & \mu_{xz}(H_{xz}) \\ \mu_{yx}(H_{yx}) & \mu_{yy}(H_{yy}) & \mu_{yz}(H_{yz}) \\ \mu_{zx}(H_{zx}) & \mu_{zy}(H_{zy}) & \mu_{zz}(H_{zz}) \end{pmatrix}, \quad (2)$$

which depends on the magnetic history of the material (hysteresis) and is locally distributed (index  $i$ ), i.e., changes its characteristics over a small region. Coordinates in Eq. (2) refer here to the Cartesian space. A linear and isotropic permeability can be written as scalar

$$x_i = \mu_i \hat{=} \begin{pmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{xx} & 0 \\ 0 & 0 & \mu_{xx} \end{pmatrix}. \quad (3)$$

The investigated experimental setup can be described by an open magnetic circuit implemented by an electromagnet (yoke, coils) that generates a nearly  $x$ -directional excitation magnetic field at the region of interest (Fig. 1). The magnetometer or gradiometer configuration of the GMR sensing areas (gray) describes separate sensors; each sensor belongs to the observation area located above the region of interest. The GMR sensing area can be modeled as magnetic field detector in the micrometer scale, and defect sizes and lift-offs have similar dimensions. The real nanoscopic structure of the sensor in  $z$ -direction can be neglected. Neither the electromagnet nor the GMR sensor is in contact with the object, which ensures non-destructive testing. The testing probe scans in the  $x$ -direction. The components of the probe (yoke, coils, GMR sensor) need to be assembled. Since our probes are assembled manually, it is likely that positioning errors arise. A small lift-off of the electromagnet couples the excited magnetic field better to the testing object and reduces additional magnetic stray-fields in the air-gap region.

The forward problem is considered for two separate regions,  $\Omega_g$  and  $\Omega$ , which allows effective calculations at different scales:

- a millimeter-scale region for the electromagnet,
- a micrometer-scale subregion for the GMR and the micro-cracks.

Each region respects its own scale [13] and a coupling is achieved via boundary conditions. Here,  $\Omega_g$  represents a three-dimensional region, and  $\Omega$  refers to a two-dimensional subregion in the  $xy$ -plane at  $z = 0$  mm (Fig. 1). A permeability distribution with linear and isotropic material parameters is applied to the material part of each region. A low excitation current allows consideration of only the linear section of the yoke and the object's nonlinear material hysteresis behavior. Then, approximation errors are expected at locations with very high, undesired magnetic field values: e.g., at the edges of the yoke.

Region  $\Omega_g$  considers the yoke, coils, object, and air with a homogeneous material parameter distribution. It serves to simulate the excitation magnetic field distribution  $\mathbf{H}_a$  at a subregion's boundary (region  $\Omega$ ) produced by the electromagnet (Fig. 1). The GMR sensor signals for the defect-free situation can be approximated via post-processing of the results. Let us start with the corresponding strong formulation of the forward problem in terms of the magnetic vector potential  $\mathbf{A}$  using the  $A$ -VA method (Vector Fields Opera, Version 15, Cobham plc, Wimborne, UK). The magnetic induction  $\mathbf{B}$  can be written as the superposition of an induction  $\mathbf{B}_s$  caused by an electric source current  $\mathbf{J}_s$ , which can be written as

$$\mathbf{B}_s = \mu_0 \mathbf{H}_s = \nabla \times \mathbf{A}_s \quad (4)$$

using the impressed vector potential  $\mathbf{A}_s$ . The source magnetic field  $\mathbf{H}_s$  can be analytically calculated by the Biot-Savart law  $\mathbf{J}_s = \nabla \times \mathbf{H}_s$ . The induction  $\mathbf{B}$  can be split into a source component  $\mathbf{B}_s$  and a material component  $\mathbf{B}_m$  (5). Let  $\mathbf{A}_r$  refer to the reduced magnetic vector potential of the non-conducting air region  $\Omega_{g,n}$  (6), and  $\mathbf{A}_m$  to the total magnetic vector potential of the conducting material region  $\Omega_{g,c}$  (7):

$$\mathbf{B} = \mathbf{B}_s + \mathbf{B}_m \quad (5)$$

$$\mathbf{B} = \nabla \times \mathbf{A}_s + \nabla \times \mathbf{A}_r, \quad \mathbf{B} \in \Omega_{g,n} \quad (6)$$

$$\mathbf{B} = \nabla \times \mathbf{A}_s + \nabla \times \mathbf{A}_m, \quad \mathbf{B} \in \Omega_{g,c}. \quad (7)$$

Hereby, the air region  $\Omega_{g,n}$  completely surrounds each simply connected material region  $\Omega_{g,c}$ . The three-dimensional region is discretized by quadratic-order tetrahedra. Application of the Dirichlet condition at the boundary of the computational region forces the magnetic vector potential to be zero (far-field boundary condition).

The region  $\Omega$  refers to a subregion of the whole computational region and its objective is to solve for defect signals  $\mathbf{y}$  and to incorporate a sensor model explicitly. It covers an observation area and the region of interest (Fig. 1). On the subregion, forward and adjoint computations can be performed for different meshes and different finite-element formulations. For a forward computation on the subregion  $\Omega$ , we focus on the strong formulation in terms of the magnetic scalar potential  $\phi$  satisfying the partial differential equation I (PDE I)

$$\nabla \cdot (\mu \nabla \phi) = 0 \text{ in } \Omega \quad (8)$$

with von Neumann boundary conditions

$$\mathbf{n} \cdot (\mu \mathbf{H}_a) = 0 \text{ on } \Gamma_n, \quad (9)$$

where  $\mathbf{H}_a$  is the magnetic field excited by the electromagnet at the boundary of the region  $\Gamma_n$ . The magnetic permeability  $\mu = \mu_0 \mu_r$  is the product of the vacuum permeability  $\mu_0$  and the relative permeability  $\mu_r$ . The finite-element analysis is formulated in terms of the total magnetic scalar potential  $\phi$ , i.e., no current flow is present. Computations regarding the subregions were performed with GetDP [14].

The source region is a sub-surface region inside the material. It is where the material parameters are investigated and the solutions represented. For this region, parametrization is used to

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