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Isogeometric shape optimization of shells using semi-analytical sensitivity analysis and sensitivity weighting

J. Kiendl [b], R. Schmidt [a], R. Wüchner [a], K.-U. Bletzinger [a,*]

[a] Lehrstuhl für Statik, Technische Universität München, Arcisstr. 21, 80333 München, Germany
[b] Dipartimento di Ingegneria Civile ed Architettura, Università degli Studi di Pavia, Via Ferrata 1, 27100 Pavia, Italy

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ABSTRACT

We present isogeometric shape optimization for shell structures applying sensitivity weighting and semi-analytical analysis. We use a rotation-free shell formulation and all involved geometry models, i.e., initial design, analysis model, optimization model, and final design use the same geometric basis, in particular NURBS. A sensitivity weighting scheme is presented which eliminates certain effects of the chosen discretization on the design update. A multilevel design approach is applied such that the design space can be chosen independently from the analysis space. The use of semi-analytical sensitivities allows having different polynomial degrees for design and analysis model. Different numerical examples are performed which confirm the applicability of the proposed method. Furthermore, a shape optimization example with an exact solution is presented which can serve as general benchmark for shape optimization methods.

1. Introduction

Shape optimization is a discipline in structural optimization which aims at finding the optimal shape of a structure with respect to a specific objective, for example, minimizing weight, strain energy, or the maximum stresses. Especially for shells, the structural behavior is crucially determined by their shape since it is the curved geometry which permits that loads are carried mainly by membrane forces reducing bending moments [10,12]. This, in turn, guarantees an optimized use of the material and a minimization of weight. Classical shell design is characterized by specific geometric classes like, e.g., cylindrical shells, spherical shells, shells of revolution in general, etc. The term "free-form shell" is used as to distinguish other shapes from those classical ones. Free-form shells include hanging forms, minimal surfaces, and other shapes which most of them are explained by mechanical principles of shape generation, also including shapes found from structural shape optimization as discussed in the present paper. Due to this definition free-form shells can be defined by spline surfaces as well. In architecture and civil engineering there exists a rich tradition dealing with the experimental as well as computational design and analysis of free-form shells, e.g., refer to the work of the International Association for Shell and Spatial Structures (IASS) for further reading [1,37].

Computational shape optimization combines the methods of mathematical optimization and computational mechanics (e.g., finite element (FE) analysis) for determining the structural responses to changes in the design. The most decisive step in defining the optimization problem is the choice of design variables, which strongly depends on the geometry parametri-

* Corresponding author. Tel.: +49 8928922422; fax: +49 8928922421.
E-mail address: kub@tum.de (K.-U. Bletzinger).

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zation of the design model. For shape optimization combined with finite element methods, there are typically two geometric models involved: the CAD (Computer-Aided Design) model and the finite element mesh. Accordingly, two different approaches for shape optimization have developed, namely the CAD-based and the FE-based approach.

In the CAD-based approach, optimization is performed on the CAD model, for example, a B-spline or NURBS (Non-Uniform Rational B-splines) parametrization of the geometry [11,13,15], and the corresponding shape parameters are used as design variables. As a consequence, the design is typically defined by a relatively low number of design variables resulting in a small design space, i.e., the space of possible shapes. Furthermore, a permanent communication between the optimization model and the finite element model is required, i.e., after each design update the FE mesh has to be regenerated.

In the FE-based approach, also called parameter-free shape optimization, the optimization is performed on the finite element model, i.e., the spatial locations of the finite element nodes are the design variables. This gives a large design space which allows finding optimal solutions which are beyond the designers experience. However, the huge design space can also create very irregular solutions, producing wiggly shapes and distorted meshes. In order to avoid this, mesh regularization and filtering techniques may be applied [9,14,24,25,29].

The isogeometric concept, as firstly proposed in [30], offers the possibility to overcome the traditional problem in shape optimization of having different geometric parametrizations involved. In isogeometric analysis (IGA), the functions from the CAD model (e.g., NURBS, T-splines, Subdivision Surfaces) are adopted as basis functions for analysis. Up to now, NURBS are the predominant technology in isogeometric analysis. Over the last years, isogeometric analysis has been applied to different areas of computational mechanics, e.g., [2,3,5,6,8,21,22,26,27,46–48], and has shown superior accuracy compared to standard FE analysis in many cases. The high regularity of the NURBS functions further allows the implementation of a geometrically and kinematically exact, rotation-free shell formulation [33], which is also used in the present paper.

Applying the isogeometric concept to shape optimization, the distinction between CAD-based and FE-based shape parametrization is redundant since both design and analysis rely on the same geometric basis. Isogeometric shape optimization has firstly been presented in [50], and since then has been further investigated in several publications which also include topology optimization [4,16,28,31,34–36,38–40,43–45,49]. In [18], a shape optimization approach for shell structures using subdivision surfaces [17] has been presented, which can also be considered as an isogeometric approach although it was published before the advent of isogeometric analysis.

In this paper, we present isogeometric shape optimization for shell structures using sensitivity weighting and semi-analytical sensitivity analysis. The proposed sensitivity weighting scheme is a correction of the design update eliminating effects arising from the specific discretization, and the use of semi-analytical sensitivity analysis allows employing different polynomial degrees for design and analysis model. The proposed method is studied on different numerical examples confirming its applicability and flexibility. Together with the numerical examples, we also present a benchmark problem for shape optimization and derive the exact solution. To the best of our knowledge, such a solution is not available in the literature. The paper is outlined as follows. In Section 2, we give a short introduction to B-splines and NURBS which are the basis for both the design and the analysis model. Section 3 briefly reviews the isogeometric shell formulation used for structural response analysis. In Section 4, we present the concept of isogeometric shape optimization and the design parametrization. In Section 5, we present an isogeometric sensitivity weighting scheme for a corrected design update. In Section 6, we study the effect of k-refinement in the context of the multilevel design concept. Section 7 provides an example on which several aspects of the presented method are demonstrated while in Section 8 a benchmark problem for shape optimization is derived analytically. In Section 9, finally, we draw the conclusions.

2. B-splines and NURBS

In the presented method, B-splines and NURBS play a central role since they serve both for the parametrization of the design model and as basis functions for isogeometric analysis.

2.1. Basis functions

B-splines are piecewise polynomials defined by the polynomial degree p and a knot vector $\Xi = [\xi_1, \xi_2, \ldots, \xi_{n+p+1}]$, where n is the number of basis functions. The knot vector is a set of parametric coordinates ξ_i which divide the parametric space into sections and define the continuity of the functions at these locations. At a single knot the B-splines are C^{p-1} continuous, if a knot is repeated k times, the continuity is reduced to C^{p-k}.

The B-spline basis functions of degree p are defined by the following recursion formula. For $p = 0$ it is:

$$N_{i,0}(\xi) = \begin{cases} 1 & \xi_i \leqslant \xi < \xi_{i+1}, \\ 0 & \text{otherwise}. \end{cases} \tag{1}$$

For $p \geqslant 1$ it is:

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi). \tag{2}$$