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Maximizing the reward in the relocation problem with generalized due dates

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ABSTRACT

The relocation problem, based on a public housing project in Boston, USA, is a generalized resource-constrained scheduling problem in which the amount of resources (new housing units) returned by a completed job (building) is not necessarily the same as the amount of resources (original housing units) it started out with for processing. In this paper we consider a variant where several generalized due dates are specified to define the number of new housing units that should be built in the entire duration of the project. Generalized due dates are different from conventional due dates in that they are job independent and common to all jobs. In the present study each generalized due date is given to specify an expected percentage of completion of the project. Given an initial number of temporary housing units, the goal is to find a feasible reconstruction sequence that maximizes the total reward over all generalized due dates. This paper investigates the time complexity of the problem. Two upper bounds and a dominance property are proposed for the design of branch-and-bound algorithms. Computational experiments are carried out to assess the efficiency of the proposed properties. The results show that the proposed properties can significantly reduce the time required for producing an optimal schedule.

1. Introduction

Resource constraints are one of the issues that are the most commonly addressed in project scheduling and management (Al-Fawzan and Haouari, 2005; Drezet and Billaut, 2008; Kobylański and Kuchta, 2007; Mingozzi et al., 1998; Pesenti and Ukovich, 2003). The study on the relocation problem arose from a public housing redevelopment project in Boston (PHRG, 1986; Kaplan, 1986). The goals of the project were to tear down some old buildings and build new ones in the same area. For this housing redevelopment scheme, the authorities provided sufficient temporary housing units for the tenants who would be evacuated from the area being redeveloped. More specifically, the authorities wanted a construction sequence of the new buildings and when and where all the displaced tenants were to be housed during the redevelopment process. This relocation problem can be looked at from an optimization point of view, in order to determine the minimum initial budget guaranteeing a feasible redevelopment sequence of the buildings. In this paper, we will consider a variant of the relocation problem that takes into account check points of the redevelopment.

In most scheduling problems, the due dates are job-dependent. That is, each due date is associated with a particular job and each individual job is expected to be completed before its corresponding due date. Hall (1986) first introduced the concept of generalized due dates. When scheduling using generalized due dates, the due dates are job independent such that a generalized due date is associated with a certain number of jobs that must be completed prior to that point in time. The idea of generalized due dates is commonly adopted in the real world. For example, a company might have a 2-year

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Notation:

$J=\{1, 2, \ldots, n\}$	set of jobs to be processed
V_0	initial resource level
p_i	processing time of job i
a_i	amount of resource (housing units) required for processing job i
b_i	amount of resource (housing units) returned at the completion of job i
D_k	generalized due date k, $k=1, 2, \ldots, m$; note that $D_{m+1}=\sum_{i=1}^{n} p_i$
h_k	expected amount of housing units to be completed by D_k
C_i	completion time of job i
V_t	resource level in the common pool at time t
B_k	cumulative reward gained at due date D_k, i.e. $B_k=\sum_{C_i \leqslant D_k} b_i$
$Z(S)$	$\sum_{k=1}^{m} B_k$, total reward of particular schedule S

contract to produce 100 units of a product with the special requirement that one batch of 40 units must be delivered within the first year.

In this paper, we focus on the relocation problem incorporating generalized due dates. Consider the original setting of the relocation problem in the housing redevelopment project. The authorities and the construction company may set several generalized due dates based upon which they coordinate their transactions, such as pay by installments, project reviews, and so on. In this paper, we define a generalized due date as the time by which an expected percentage of the project is to be completed. For example, 50% of the newly built capacities of the project must be completed within the first year, and the entire project shall be completed at the end of the second year. At each generalized due date, if the actual percentage of the completion of the project is less than that is expected, the authorities may reduce the amount paid to the construction company as a penalty for the delay. On the other hand, it increases the amount paid as a reward for a more rapid progress. Such type of contract is quite common in the real world (Lock, 1996). For both theoretical and practical considerations, our study will investigate this situation.

This paper is organized into seven sections. In Section 2, we present a formal definition of the relocation problem with generalized due dates. This is followed by a literature review on the relocation problem and generalized due dates in Section 3. In Section 4, we present a proof of NP-hardness for the problem considered. Section 5 is dedicated to the design of two upper bounds and one dominance rule for developing a branch-and-bound algorithm. The computational experiments and numerical results are given in Section 6. Finally, we draw our conclusions in Section 7.

2. Problem formulation

In this section, we present a formal description of the problem under study. Relevant literature on the subject will also be addressed. Please note that throughout this paper, job and building, and number of housing units and amount of resource will be used interchangeably.

Formally, at time zero there is a common pool of V_0 units of single-type resource (housing units in the relocation problem), and a set of jobs (buildings in the relocation project) $J=\{1, 2,\ldots, n\}$ is to be processed on a single machine. Job $i \in J$ has three integer parameters: processing time p_i, amount of resource required a_i, and amount of resource returned b_i. At any time t, job i can only be considered for processing if the resource level V_t in the common pool is no less than a_i, i.e. $V_t \geqslant a_i$. When job i starts processing, it acquires and immediately consumes a_i units of resource and thus reduces the resource level in the resource pool by a_i. Upon its completion, job i immediately returns b_i units of resource back to the resource pool. No preemption is allowed, and at any moment the machine can process at most one job. In the housing redevelopment project, a_i and b_i, respectively, correspond to the numbers of housing units of building i before and after the redevelopment. The processing of job set J is associated with m generalized due dates (abbreviated as gdd hereafter) $D_1 \leqslant D_2 \leqslant \cdots \leqslant D_m$ such that each D_k is an integer belonging to the interval $[p_{\min}, P]$, where $p_{\min}=\min\{p_i | i \in J\}$and $P=\Sigma_{i \in J} p_i$. Each D_k is associated with a threshold or a number of new housing units, h_k, which are expected to be completed by D_k. Given a feasible construction sequence, we denote B_k as the total number of new housing units completed by due date D_k. If B_k is smaller than h_k, then the contractor will be penalized by a cost linearly dependent on h_k–B_k. On the other hand, if the construction progresses well and more units have been completed than expected, then a reward that is linearly dependent on B_k–h_k will be earned. We assume the unit penalty and unit reward to be the same. When the difference B_k–h_k is negative, the reward is then interpreted as a penalty. Given a certain amount of initial resource, there could be many feasible schedules of a given job set. The goal of the problem is to find a feasible schedule such that the sum of rewards over generalized due dates, $\sum_{k=1}^{m}(B_k-h_k)$ is maximized. Because $\sum_{k=1}^{m} h_k$ is fixed once the input is given, the problem is equivalent to the maximization of $\sum_{k=1}^{m} B_k$. For notational convenience, we use the standard three-field notation (Graham et al., 1979) $1|\text{rp, gdd}|\Sigma B_k$ to specify the problem of interest. The first field indicates that the problem environment is a single machine. The second field specifies special restrictions on the problems and on the jobs. It shows that the gdd version is considered in the relocation problem (rp). The third field indicates that the objective is the optimization of the cumulated reward gained over all generalized due dates.

Note that when m gdds are specified we do not include the one where the project is completely finished because the total processing length P is fixed once the input is given. Following the convention in the literature on scheduling, we use indices enclosed within brackets to