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Tsallis' non-extensive free energy as a subjective value of an uncertain reward

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ABSTRACT

Recent studies in neuroeconomics and econophysics revealed the importance of reward expectation in decision under uncertainty. Behavioral neuroeconomic studies have proposed that the unpredictability and the probability of an uncertain reward are distinctly encoded as entropy and a distorted probability weight, respectively, in the separate neural systems. However, previous behavioral economic and decision-theoretic models could not quantify reward-seeking and uncertainty aversion in a theoretically consistent manner. In this paper, we have: (i) proposed that generalized Helmholtz free energy in Tsallis' non-extensive thermostatistics can be utilized to quantify a perceived value of an uncertain reward, and (ii) empirically examined the explanatory powers of the models. Future study directions in neuroeconomics and econophysics by utilizing the Tsallis' free energy model are discussed.

1. Introduction

Humans and non-human animals devalue probabilistic rewards as the receipt becomes more uncertain. The preference for a certain reward over an uncertain reward of an equal expected value is referred to as risk aversion in decision-making under risk/uncertainty. Neuropsychopharmacological studies reported that several types of neurochemical substances such as nicotine and serotonin dramatically modulate human decision under uncertainty [1,2]. In microeconomic theory, risk aversion is represented by a concave utility function [3]. Decision-making under uncertainty has been drawing much attention in behavioral neuroeconomics, econophysics and neurofinance, because departures from the prediction of the microeconomic theory (i.e., anomalies) have repeatedly been demonstrated in human choice behavior [4–7]. To establish quantitatively precise models of actual human decision-making under risk is important for understanding financial markets, risky decision by substance abusers and pathological gamblers [1,8]. Notably, neuroimaging studies have identified neural activities associated with uncertainty in decision under risk [9,8,10].

Empirical studies in behavioral neuroeconomics on decision-making under risk and uncertainty have reported the following important findings:

(A) People overweight small probabilities and underweight large probabilities [11];
(B) People have aversion to "ignorance" on the outcomes of uncertain rewards (i.e., people prefer a predictable outcome to an unpredictable one) [4].

It is important to note that von Neumann–Morgenstern's traditional expected utility theory [3] cannot predict/explain these psychological tendencies observed in humans. In order to explain and formalize anomaly (A), the prospect theory (PT) has

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been proposed [11]. In PT, it is assumed that a probability of an uncertain reward is nonlinearly transformed/distorted into a psychophysical "probability weight" function [5,6] which is concave in probability at small probabilities and convex at large probabilities. Anomaly (B) has most dramatically been demonstrated in the Ellsberg paradox experiment in which people prefer uncertain rewards with a known probability distribution to an unknown probability distribution [4]. In order to quantify human aversion to ignorance on outcomes with known probabilities, Shannon entropy has been introduced [12]. However, to date, little effort has been spent on unifying PT and Tsallis thermostatistics-based decision theory, and combining the psychophysical and information-theoretic factors in decision under risk. It is to be noted that Antenedo, Tsallis & Martinez (2002) [13] is a pioneering investigation into this direction. Cajueiro (2006)'s attempt [14] to apply Tsallis' statistics-based deformed algebra to intertemporal choice and empirical estimation of parameters in the model [15] are also in a similar line to the present study.

This paper is organized in the following manner. In Section 2, I briefly introduce PT and the role of entropy in decision under risk, in Section 3, I explain that a generalized free energy in Tsallis' non-extensive thermostatistics can be utilized as a subjective value of an uncertain reward, and in Section 4, an experimental comparison between the previously-proposed entropy model and the present Tsallis' generalized free energy models is reported. Finally, in Section 5, I suggest some conclusions from this study and future study directions by utilizing the present Tsallis' free energy model.

2. Psychophysics and information theory of decision under uncertainty

Suppose the lottery $L(x_1, p_1; \ldots; x_n, p_n)$, where $x_i > 0$ (gain) occurs with probability p_i (an integer i satisfies $0 < i < n + 1$). PT assumes that a subjective value ("prospect") $V(x, p) = V(x_1, p_1; \ldots; x_n, p_n)$ of an uncertain reward is equal to $\Sigma_i v(x_i)w(p_i)$, where $v(x_i)$ is a subjective value of a certain reward x_i when the probability of receipt $p_i = 1$ and $w(p_i)$ is a probability weight function which reflects a psychophysical distortion in the perception of probability values [5,6,11]. Let us consider a simple example, the Bernoulli-type lottery $L(x, p; 0, 1 - p)$. It is to be noted that in the Bernoulli-type lottery, the outcomes consist of a gain x with probability p and a non-gain with probability $(1 - p)$; in other words, $p_1 = p$ and $p_2 = 1 - p$. The rationale for considering the Bernoulli-type lottery here is that most experimental work on human probabilistic choice has been focused on this type of lottery, in order to elucidate psychophysical and neural processes underlying human decision-making under uncertainty [1,6,7].

In PT [11], a subjective value of the Bernoulli-type lottery is expressed as: $V(x, p) = \Sigma_{i=1,2} v(x_i)w(p_i) = v(x)w(p)$, because $x_1 = x, x_2 = 0, p_1 = p, p_2 = 1 - p$, and $v(0) = w(0) = 0$. Note that $w(p)$ is a monotonically increasing function of probability p. By assuming that $\mathrm{d}^2 w(p)/\mathrm{d}p^2 < 0$ for small probabilities (typically for $p < 1/e$, according to Prelec (1998)'s proposal [5]) and $\mathrm{d}^2 w(p)/\mathrm{d}p^2 > 0$ for large probabilities (an inverted-S-shape probability weight [11]), PT can capture one of the human biases in decision under uncertainty; i.e., overweighting of small probabilities and underweighting of large probabilities. Behavioral economic studies have proposed several functional forms of the probability weight function. For instance, Gonzalez and Wu (1999) proposed $w(p) = \delta p^\gamma/(\delta p^\gamma + (1 - p)^\gamma)$ [6] and Prelec [5] proposed $w(p) = \exp(-\delta(-\log(p))^\gamma)$, where δ and γ are positive free parameters. Irrespective of the functional forms of the probability weight, the nonlinear distortion of probability p into $w(p)$ may result from a psychophysical effect on the perception of a probability [5,6,11,12]. In Antenedo et al.'s econophysical study [13] utilizing Tsallis' generalized thermostatistics and Takahashi's entropy model [12], the probability weight has been assumed to be a q-probability (i.e. an escort probability in Tsallis non-extensive thermostatistics [16,17]). It is to be noted here that although PT [11] can describe anomaly (A) in Section 1, PT has not originally been proposed for the account of ambiguity/unpredictability aversion. A recent neuroimaging study on human decision-making under risk reported psychological processes of a reward expectation and aversion to unpredictability on uncertain rewards are distinctly represented in the brain [10]. Because an original PT describes subject's reward expectation ("prospect") [11], these findings require the modification of traditional PT.

In order to quantify the aversion to ignorance/unpredictability on outcomes of the lottery $L(x, p; 0, 1-p)$, Takahashi [12] introduced Shannon's information-theoretic entropy as a parameter of ignorance on probabilistic outcomes. In the entropy model, a subjective value of the Bernoulli-type uncertain reward is $V(x, p) = v(x)p^a - TS_{\mathrm{Shannon}}$, where free parameter a indicates the psychophysical effect on the perception of a small probability, $S_{\mathrm{shannon}} = -\Sigma_i p_i \log p_i = -p \log p - (1 - p)\log(1 - p)$ is a conventional Shannon entropy in information theory, and free parameter T indicates a subject's degree of unpredictability aversion ($v(x) = x$ was assumed in [7]). When $t := T/v(x)$ is introduced, the subjective value can be expressed as $v(x)(p^a - tS_{\mathrm{Shannon}})$ and $(p^a - tS_{\mathrm{Shannon}})$ corresponds to the probability weight in PT [11]. It is to be noted that the parameter of ignorance/unpredictability S_{shannon} is maximal at $p = 0.5$ and minimal at $p = 0$ or 1. A recent behavioral economic study [7] demonstrated that the entropy model fit human subjects' probabilistic choice behavior at the group level, better than a simple hyperbolic probability-discounting model (a model often utilized in neuropsychopharmacology) [1]. Also, a recent neuroeconomic study utilizing a psychologically similar model reported that unpredictability largest at $p = 0.5$ in probabilistic choice was associated with the activation of brain regions such as the insula (a neural circuit for disgust) [10]. It can be seen that, as in Antenedo et al.'s model [13] based on Tsallis' statistics, the previously-proposed entropy model assumes that the probability weight function is q-probability in the Tsallis statistics-based framework. However, no study to date applies Tsallis' thermostatistics-based generalized free energy to decision under risk in a theoretically consistent manner. Antenedo et al.'s study [13] also had a limitation that the model was not allowed to describe agents' inverted-S-shaped probability weight function.