Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 276 (2014) 212–232
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Sensitivity analysis of the scaled boundary finite element method for elastostatics

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Received 20 November 2013; received in revised form 14 March 2014; accepted 14 March 2014
Available online 3 April 2014

Abstract

As a semi-analytical structural analysis algorithm, the scaled boundary finite-element method (SBFEM) only discretizes the boundary of the analyzed domain without the need of fundamental solutions, which makes it powerful for problems with stress singularity or unbounded foundation media. In this paper, a sensitivity analysis method of SBFEM is proposed for elastostatics, through which the first order derivatives of the structural responses with respect to the design parameters can be obtained efficiently and accurately. An approach is suggested to compute the eigenvalue and eigenvector sensitivities of the Hamilton matrix, which are then used to calculate the analytical derivative of the stiffness matrix. Based on these calculations, the sensitivities of displacements and stresses are further obtained by a series of differential equations. The proposed sensitivity analysis method is also applied to the fracture mechanics problems. Three numerical examples are investigated to demonstrate the validity of the proposed method.

Keywords: Sensitivity analysis; The scaled boundary finite element method; Eigensensitivity analysis; The stress intensity factor

1. Introduction

The scaled boundary finite-element method (SBFEM) is a semi-analytical method for structural analysis which was proposed by Song and Wolf [1,2]. For simple geometries, it only requires meshing on the boundary of the analyzed domain in comparison to the finite element method (FEM) and does not need a fundamental solution in comparison to the boundary element method (BEM). For more complex problems, the interior of the domain can be discretised into subdomains to satisfy the scaling requirement [3,4]. In SBFEM, stress singularities at cracks, corners and bi-material interfaces are analytically represented so that the accurate stress

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http://dx.doi.org/10.1016/j.cma.2014.03.005

intensity factors (SIFs) can be calculated directly by definition with no singular integrals. In addition, general anisotropic material can be analyzed by SBFEM without any increase in computational effort. Therefore, SBFEM has been successfully used in many research areas, such as the soil structure interaction problems, wave interaction problems, dynamic interaction of structure with unbounded foundation media, and especially the fracture mechanics problems.

In recent years, many researchers have studied the SBFEM. Wolf [5] extended the SBFEM to analyze the response of unbounded soil. Schauer et al. [6] introduced a parallel algorithm for 3-D soil–structure interaction in time domain by combining FEM with SBFEM. Bransch and Lehmann [7] developed a method for the calculation of elastic–plastic building ground deformations and elastic–plastic building ground failure with a coupled FEM/SBFEM approach. Birk and Behnke [8] derived a modified SBFEM for the analysis of 3D-layered continua based on the use of a scaling line instead of a scaling centre. Tao et al. [9] and Liu et al. [10,11] extended the SBFEM to solve short-crested wave interaction with a cylindrical structure. Song and Tao [12] developed an efficient SBFEM approach to analyze the wave interaction with a non-uniform porous cylinder by introducing a porous-effect parameter. Ekevid and Wiberg [13] analyzed the wave propagation related to high-speed train for unbounded domains by SBFEM. Liu et al. [14] applied the SBFEM to analyze the quadruple corner-cut ridged square waveguide. He et al. [15] developed a novel method for cyclically symmetric two-dimensional elastic analysis based on SBFEM. Song and Wolf [16] derived a semi-analytical solution of the singular stress occurring at cracks in anisotropic multi-materials by SBFEM. Song [17] and Yang et al. [18] studied the dynamic stress intensity factors by SBFEM. Yang and Deeks [19] and Ooi and Yang [20] extended the SBFEM to the crack propagation problem. Shi et al. [21] analyzed the crack propagation of gravity dams using the polygon scaled boundary finite method. For the advantages that only the boundaries need to be discretized and a large range of perturbation of the crack tip location can be modeled without re-meshing, Chowdhury et al. [22,23] studied the effect of uncertainty in the crack geometry on the reliability of cracked structures using SBFEM. Chen et al. [24] studied the identification inverse problem by SBFEM. Jiang et al. [25] developed a probability-interval hybrid reliability analysis method for cracked structures based on SBFEM.

In more recent years, Gravenkamp et al. [26] presented an approach to compute dispersion curves of elastic waveguides based on SBFEM. Liu and Lin [27] extended the SBFEM to solve electrostatic problems. Man et al. [28] developed a 3D-consistent technique with the through-thickness solutions represented analytically to analyze piezoelectric plates using the SBFEM. Vu and Deeks [29] used fundamental solutions in the scaled boundary finite element method to solve problems with concentrated loads. Man et al. [30] presented a technique without numerical shear locking issue for plate bending analysis based on the scaled boundary FEM. Ooi et al. [31] developed an efficient methodology for automatic dynamic crack propagation simulations using scaled boundary polygon elements. Li et al. [32] presented a technique to analyze two-dimensional fracture problems of piezoelectric materials based on SBFEM. He et al. [33] developed an element-free Galerkin scaled boundary method for solving steady-state heat transfer problems.

Though some important progresses have been made in the above work, the existing methods are nearly all focused on the structural response analysis through SBFEM, while less work has been conducted for its sensitivity analysis. To real structures, many parameters such as the material properties, loads and geometrical characteristics, may have different degrees of effects to the structural responses and thereby the structural performance. It is very important for structural analysis and design to identify which parameters have effects to the responses and sometimes which one has the largest effect, which seems the principal reason that we need to develop the sensitivity analysis technique. Additionally, the sensitivity information is also very important in some other fields, such as structural optimization design, reliability analysis, etc. For the importance of sensitivity analysis, it has been studied by many authors based on FEM (e.g. [34,35]). Recently, the numerical difference method [22–24] was employed to compute the sensitivity for SBFEM, however, it will involve two times of SBFEM computations in a sensitivity analysis. Furthermore, the selection of a proper step is also a problem for the difference method. Thus to develop an efficient and robust semi-analytical sensitivity analysis technique is very necessary for SBFEM for improving its applicability in real structural design.

This paper aims to propose a semi-analytical sensitivity analysis method for elastostatics based on SBFEM, through which the gradients of the structural responses with respect to some important design parameters can be efficiently and accurately computed. The remainder of this paper is organized as follows. Section 2 briefly