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Adjoint shape design sensitivity analysis of fluid–solid interactions using concurrent mesh velocity in ALE formulation

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ABSTRACT

A coupled variational equation for fluid–solid interaction (FSI) problems is derived using a steady state Navier–Stokes equation for incompressible flows, an equilibrium equation for geometrically nonlinear solids, a traction continuity condition at interfaces, and a pseudo-equilibrium equation for mesh velocity. The moving boundary in arbitrary Lagrangian–Eulerian (ALE) formulation is included in the variational equations by the mesh velocity obtained from a displacement-loaded pseudo-structural problem at a concurrent configuration, which eventually facilitates to derive shape design sensitivity. A continuum-based adjoint shape sensitivity is derived under ALE formulation, which turns out to be very accurate and efficient due to the utilization of converged tangent and the linearity of both adjoint and sensitivity equations. Through numerical examples, the obtained sensitivity is verified in terms of accuracy and efficiency compared with finite difference sensitivity and further applied to the shape optimization problem of finding a stiff structure while satisfying a volume constraint.

1. Introduction

In many engineering disciplines such as aerospace, marine, automotive, and wind engineering fields, the consideration of FSI systems is substantial for quality engineering analysis and thus its necessity is continuously increasing [1]. Nevertheless, the prohibitive amount of computations has been one of the major issues in the coupled FSI analysis. When intended to design optimization for the large scale transient dynamic problems, it is impractical to employ finite difference sensitivity to perform shape design optimization of the coupled FSI problems. Furthermore, the inaccuracy of finite differencing could result in either convergence difficulty or premature results during the design optimization [2].

For the kinematical description of fluid domain, the difficulty of severe mesh distortion is inevitable if a Lagrangian approach is employed. On the other hand, the mesh distortion can be handled easily in an Eulerian approach. However, the treatment of moving boundaries and interfaces is challenging. A unified Lagrangian–Eulerian kinematical description of fluid domain is developed by Hughes et al. [3] so that the grid points are displaced independently of fluid motion, where the moving boundary is described by the movement of the reference frame. Several algorithms have been proposed to control the movement of fluid mesh. An ALE kinematical description of fluid domain is adopted [4] so that grid points can be displaced independently of fluid motion. They imposed the mesh velocity by averaging grid velocities of neighboring nodes at the previous time step. For the shape sensitivity analysis in updated Lagrangian [5] or ALE formulation, the design velocity field needs to be updated at every configuration [6, Section 2.3]. *In this paper, we employ a moving mesh method where the domain is considered as an elastic body subjected to a prescribed motion on its boundary.*

To handle the moving boundary and interface of FSI problems, Farhat et al. [7] developed a non-matching discretization method that uses different grids to analyze two fields allowing the incompatibility at interfaces. In shape design optimization, the approximation to obtain the geometric information such as tangent and normal could result in inaccuracy issues in transferring the load and motion of boundary at each time step. Chiandussi et al. [8] proposed a mesh update method based on this approach. The mechanical properties of elements are properly selected to minimize the mesh distortion. Yoshida and Kawahara [9] performed a shape optimization using ALE FEM. The oscillating motion of rigid body was expressed by mesh displacements. They obtained the mesh velocity using an iterative scheme. Also, a space–time Piola transformation method [10] is developed to treat the compatible interfaces, where the equations of motion are described in a reference domain. Recently, using a NURBS-based

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isogeometric analysis method that preserves geometrical exactness for the spatial discretization, Bazilevs et al. [11] present a fully coupled monolithic formulation of the FSI problems on a moving domain with nonlinear hyperelastic solids. Heinrich et al. [12] performed a shape design optimization of FSI problem which is solved by using the isogeometric approach. They used a gradient-free optimizer due to the expensive cost for the DSA.

In the early stage of theoretical development for the coupled FSI problems, many research efforts have been devoted to iterative methods [13] where both disciplines are solved independently in a staggered way at each time step. When it is extended to complicated nonlinear problems, however, some difficulties in convergence and efficiency could occur. To overcome these difficulties, Ghattas and Li [14] proposed a fully coupled method that solves two fields at the same time, using a unified tangent matrix that shares the same solution space for a traction continuity equation at interfaces. The surface traction expressed in terms of 1st Piola–Kirchhoff stress at the undeformed interface is identical to the surface traction expressed in terms of the Cauchy stress at its image on the deformed interfaces. Also, a domain decomposition method is used for the sensitivity analysis of nonlinear aeroelasticity [15]. In a similar way, Lund et al. [16] performed the shape design optimization of FSI problems using semi-analytical sensitivity. Fernandez and Moubachir [17] proposed a shape design sensitivity analysis method and the corresponding linearized equation using ALE formulation and direct differentiation method for aeroelastic systems. Monolithic ALE framework is also used in deriving adjoint shape sensitivities for the goal-oriented adaptive mesh-refinement [18,19]. *In this paper, to obtain efficient and accurate shape sensitivity for the coupled FSI problems, we derive continuum-based shape sensitivity. The obtained adjoint shape sensitivity is utilized to develop a shape optimization method.*

In Section 2, considering a Navier–Stokes equation for incompressible fluids, an equilibrium equation for geometrically nonlinear solids, and traction continuity conditions at interfaces, a coupled variational equation for the FSI problems is derived and solved using the FEM and Newton–Raphson scheme. Second Piola–Kirchhoff stress and Green–Lagrange strain tensors are used to handle the finite deformation of solids in total Lagrangian formulation. A no-slip condition is imposed at the interfaces: the surface traction expressed in terms of the first Piola–Kirchhoff stress at the undeformed interfaces is assumed to be identical to the one expressed in terms of the Cauchy stress at its image on the deformed interfaces [14].

In Section 3, an adjoint shape DSA method based on material derivative concept [20] is developed for steady state FSI problems. Even though the cost is very expensive for the response analysis due to the nonlinear nature of problems, the computation cost is trivial for the DSA since the converged solution and the tangent in the response analysis are readily available [21]. Using the developed DSA method, a shape optimization method is formulated for the FSI problems. The objective is to either maximize the stiffness of solids or minimize the drag on the solids while satisfying a constraint of allowable volume. Traditionally, the design and analysis are performed iteratively to produce an optimal design. However, in recent years, the integration of CAD, analysis, and design optimization become more important [22]. Thus, the whole domain including fluids, solids, and interfaces is parameterized to facilitate the shape variations using NURBS patches whose control points are considered as design variables. Akbari et al. [23] showed that if the same discretization, numerical integration, and linear design velocity fields as used in the response analysis are employed, the sensitivities obtained from the continuum–discrete and the discrete–discrete approaches are theoretically equivalent.

In Section 4, through numerical examples, the accuracy and efficiency of the developed DSA method is compared with finite difference sensitivity. The critical issue in a shape optimization of FSI problems is the computing cost for the analysis of coupled problems that include nonlinearity in each discipline. Thus, finite difference sensitivity is not practical for the shape design optimization, compared to a fully analytical one. However, the developed shape DSA method is extremely efficient and shown to be applicable to the FSI problems. Together with the shape parameterization method, the developed DSA method is applied to the shape optimization of FSI problems. The optimization process is shown to be very efficient and yields very reasonable results in a physical point of view.

2. FSI analysis in ALE formulation

2.1. Variational formulation

Consider a continuum body composed of solids and fluids, which is described by multiple domains and boundaries as shown in Fig. 1. The subscripts F, S, I, and I_0 in the domain Ω and boundary Γ are used to denote the fluid, solid, deformed, and undeformed interfaces, respectively. The superscripts n and d in the boundary Γ are used to denote the Neumann and Dirichlet types, respectively. The whole domain Ω is composed of $\overline{\Omega}_F$ and $\overline{\Omega}_S$ which are respectively the complete domains of fluids and solids, and they share an interface boundary Γ_{I_0} as

$$\overline{\Omega}_F = \Omega_F \cup \Gamma_F \cup \Gamma_{I_0} \tag{1}$$

and

$$\overline{\Omega}_S = \Omega_S \cup \Gamma_S \cup \Gamma_{I_0}. \tag{2}$$

In isothermal incompressible flows, an incompressibility condition is written as

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega_F \tag{3}$$

and the conservation of linear momentum is described in the ALE formulation as

$$\nabla \cdot \boldsymbol{\sigma}_F + \mathbf{b}_F = \rho_F(\mathbf{u} \cdot \nabla)\mathbf{u} - \rho_F(\mathbf{v} \cdot \nabla)\mathbf{u} \quad \text{in } \Omega_F, \tag{4}$$

where $\boldsymbol{\sigma}_F$, $\mathbf{b}_F$, ρ_F, $\mathbf{u}$, and $\mathbf{v}$ denote the Cauchy stress, body force intensity, fluid density, fluid velocity, and mesh velocity in the ALE formulation, respectively [3,4]. The constitutive relation of Newtonian fluids and the strain–velocity relation are given by

$$\boldsymbol{\sigma}_F = -p\mathbf{I} + 2\mu_F\mathbf{d} \tag{5}$$

and

$$\mathbf{d} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T), \tag{6}$$

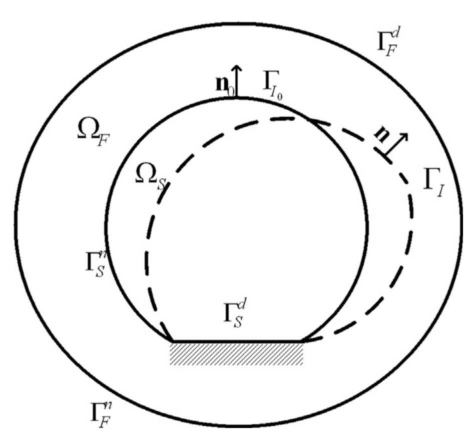

Fig. 1. Problem of fluid–solid interaction.