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The sensitivity analysis of a geometrically unstable structure under various pulse loading

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ABSTRACT

The dynamic response of a structure depends on both the structure and the applied load. A pulse approximation method proposed by Youngdahl has been proved to be applicable to stable structures and it has been adopted to deal with arbitrary loading pulse in analyses. However, for the structures possessing unstable load–displacement properties, the applicability of pulse approximation method needs to be validated.

In our previous theoretical study, by employing the rigid-perfectly plastic idealization, the static and dynamic responses of tensor skin are obtained, revealing its softening (*i.e.* geometrically unstable) behavior. In the present paper, a similar theoretical approach is employed together with the pulse approximation method to examine the effect of various pulse shapes on the final deflection of the tensor skin. It is found that the final deflections are insensitive to pulse shape, and the differences resulted from different pulses are less than 10%. Hence, the pulse approximation method is also applicable to geometrically unstable structures such as tensor skin.

Besides, for various pulse shapes, it is found that the final deflection of tensor skin increases with the total impulse as well as the effective pressure. To achieve a specified deflection, it only requires smaller total impulse for a more intensive impact loading. Furthermore, a damage diagram of tensor skin is constructed in terms of the failure impulse and the effective pressure under different pulse loading. By comparing various pulses with the same impulse level and effective pressure, the explosive pulse is more dangerous than other pulse shapes in producing failure of tensor skin.

1. Introduction

On top of a static analysis, a dynamic analysis is even more important for a structure under impact or explosive loading conditions. In general, the dynamic response of a structure is an interaction process between the structure and the applied load. In the theoretical approach of dynamics response, both the structure and the applied load have to be idealized in order to make the analysis feasible.

The rigid-perfectly plastic idealization is often adopted to predict the structural response if plastic deformation dominates the structural response [1]. For a given deformation mechanism, which is a kinematically admissible field, an upper bound of the collapse load can be obtained by energy method. For traditional structures, such as beams and plates under transverse loading, the load–displacement curves are stable; that is, the collapse load monotonically increases with displacement [1]. However, some structures exhibit unstable characteristics in their load–displacement relationship. In these cases, the load to initiate the plastic deformation is high, but the resistance of the structure decreases with the increase of its plastic deformation, so its load–displacement curve displays a kind of *softening effect*. For example, tensor skin is a kind of composite sandwich structure developed to improve the crashworthiness of helicopter subjected to water impacts [2]. From a rigid, plastic analysis of tensor skin [3], it is found that after the initial collapse, the static critical pressure first decreases then increases with the increasing central deflection. Hence, the tensor skin is a kind of unstable structure due to the nonlinear geometrical behavior in large deformation. The dynamic response of this tensor skin under water impact is of interest of the present study.

To find out an accurate dynamic structural response requires the knowledge of the actual loading pulse. In the stage of structure

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design, however, simple pulse shapes such as rectangular pulse or triangular pulse are usually adopted in the analysis; while in reality various impact conditions may produce more complex-shaped pulses. Therefore, the influence of the pulse shape on the structure response should be studied. The earliest study about the effect of pulse shape can be traced back to 1953 when Symonds [4] found that the final deflection of a free beam subjected to a concentrated force pulse only depends on the total impulse I and peak load P_{max} of the pulse. Later, Hodge [5] remarked that this conclusion was valid only for loading intensities far beyond the yield load; otherwise this simplification (by counting I and P_{max} only) may introduce a large error.

In order to eliminate pulse shape effects, two correlation parameters have been proposed by Youngdahl [6,7]. From his study, the dynamic response of a structure under a general loading pulse can be approximated by that under a rectangular pulse impulse I_e, with an effective load P_e and pulse duration $2t_{mean}$, see more details in Section 2. A number of studies by other researchers [8–12] further confirmed that the pulse approximation method is able to eliminate pulse shape effects on the dynamic plastic bending response of various structural members, such as beams, circular plates and cylindrical shells. Later, it is found [13] that when damping effect is introduced, the pulse shape effect on the maximum deflection of a simply supported beam can be eliminated by introducing I_e and P_e. A theoretical foundation for rigid-plastic responses of common structural members was established using the bound theorems and it also reveals that Youngdahl's empirical estimation for the structural response time is, in general, a lower bound on the actual response time [14].

It is worth noted that, in the previous studies of the pulse approximation method, the structures concerned are all stable ones without any softening property. Hereby, an interesting question is raised: is the pulse approximation method also applicable to unstable structures, such as the tensor skin?

In the present paper, the dynamic responses of the geometrically unstable structure, tensor skin, under various pulses will be investigated. First, the pulse approximation method and the theoretical model of tensor skin will be reviewed in Sections 2 and 3, respectively. Then the applicability of the pulse approximation method for tensor skin will be validated in Section 4. In Section 5, the effects of the total impulse and the effective pressure on the final displacement are analyzed. Finally, a damage diagram of the tensor skin is constructed for different pulse shapes in Section 6. By taking the tensor skin as a typical example, it is concluded that the pulse approximation method is also applicable to the geometrically unstable structures.

2. The pulse approximation method

Youngdahl [6] first proposed that the effect of a general loading pulse on a structure's response could be approximately equivalent to that of a rectangular pulse impulse I_e with an effective load P_e and pulse duration $2t_{mean}$, which are defined by:

$$I_e = \int_{t_y}^{t_f} P(t)\cdot \mathrm{d}t,\ \ P_e = I_e/2t_{mean},\ \ t_{mean} = \frac{1}{I_e}\int_{t_y}^{t_f} (t-t_y)P(t)\cdot \mathrm{d}t \qquad (1)$$

where $P(t)$ is the non-negative pulse magnitude varying with time t, t_y and t_f are the time instant when plastic deformation begins and ends, respectively. The final deflection of the structure in concern is determined by I_e and P_e for a general pulse load. Since both I_e and P_e depend on the integral of the general pulse, they are not very sensitive to the actual pulse shape. The method reduces the influence of pulse shape and allows a general pulse to be approximately represented by only two parameters. However, in actual applications it is still difficult to determine t_y and t_f. Hence, Youngdahl also suggested the following relation,

$$\left(t_f - t_y\right)P_y = \int_{t_y}^{t_f} P(t)\cdot \mathrm{d}t \qquad (2)$$

where P_y is the static yield load (*i.e.*, the static collapse load) of the structure. Youngdahl [7] has shown that the maximum final deflection y_{max}, depends on I_e and P_e, *i.e.*,

$$y_{max} = I_e^2 G(P_e) \qquad (3)$$

where G is a function depending on the structural configuration. Using Eqs. (1)–(3), we are able to eliminate pulse shape effects on the dynamic plastic bending response of widely-used structural members, such as beams, circular plates and cylindrical shells [9,10].

According to the pulse approximation method, if the general loading pulse satisfies Eq. (1), the corresponding response should be the same as that under the equivalent rectangular pulse. By comparing the final deflections obtained from this approximate method and from the known rigid-plastic solutions, it is found that the deviations caused by the pulse approximation method are within 15% for those stable structures [4,8].

On the other hand, it is noted that tensor skin is a typical geometrically unstable structure, for which the pressure to initiate its plastic deformation is high, but then the resistance will decrease as deformation proceeds. Hence, in this paper the applicability of the pulse approximation method for unstable structures will be investigated by analyzing the tensor skin.

In order to study the pulse shape sensitivity of tensor skin as a typical geometrically unstable structure (see Section 3 for the structural description), six pulses will be studied in this paper. As shown in Fig. 1, they include rectangular pulse, linear decreasing pulse, explosive pulse, cosine pulse, linear increasing pulse and triangular pulse. At time $t = 0$ the initial force magnitude is supposed to be greater than the initial plastic collapse load of the structure,P_0. Therefore, plastic deformation will start at $t = 0$; *i.e.*, $t_y = 0$ in Eq. (1). Here, a rectangular pulse is chosen as a reference,

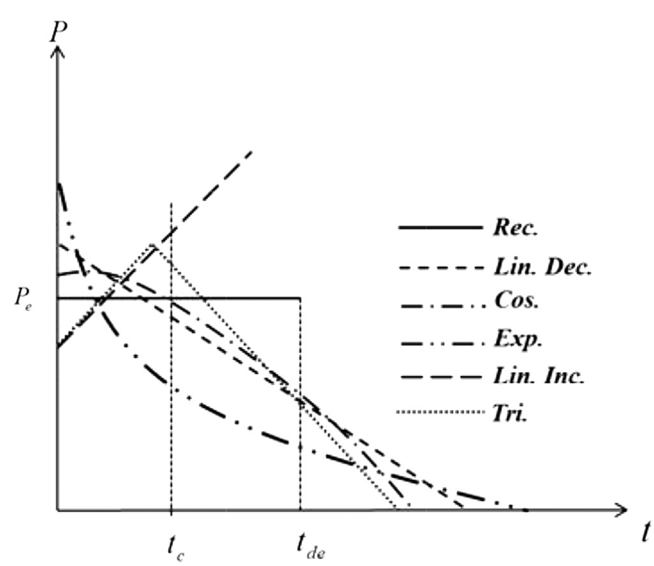

Fig. 1. Various pulses approximated by a rectangular pulse.