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Cyclic pseudo-downsampled iterative learning control for high performance tracking ☆

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ABSTRACT

In this paper, a multirate cyclic pseudo-downsampled iterative learning control (ILC) scheme is proposed. The scheme has the ability to produce a good learning transient for trajectories with high frequency components with/without initial state errors. The proposed scheme downsamples the feedback error and input signals every m samples to arrive at slower rate. Then, the downsampled slow rate signals are applied to an ILC algorithm, whose output is then interpolated and applied to an actuator. The main feature of the proposed scheme is that, for two successive iterations, the signal is downsampled with the same m but the downsampling points are time shifted along the time axis. This shifting process makes the ILC scheme cyclic along the iteration axis with a period of m cycles. Experimental results show significant improvement in tracking accuracy. Additional advantages are that the proposed scheme does not need a filter design and also reduces the computation and memory size substantially.

1. Introduction

Currently, tracking accuracy requirements in many areas have come down to the nano- or micro-meter level. Due to modeling uncertainties and disturbances, feedback control design alone is certainly not enough. Iterative learning control (ILC), which was motivated by the growth of robots performing the same task repeatedly in the mid-eighties (Arimoto, Kawamura, & Miyazaki, 1984; Middleton, Goodwin, & Longman, 1989), becomes a simple and efficient solution to either improve tracking accuracy or remove the noise/disturbance. Though different from feedback control, ILC provides a feedforward control to the system. ILC improves the tracking performance by updating the input to the system based on the tracking error in previous iterations and, therefore, is suitable for most industrial systems that are repetitive in nature.

However, a limitation of ILC is that the learning transient, or the decay of tracking error along the iteration axis, is often not monotonic. In the original work of Arimoto et al. (1984), the convergence of ILC is proven in the sense of the λ-norm. The definition of the λ-norm for a function $f:[0,T]\rightarrow\mathbb{R}^n$ is given by $\|f\|_\lambda \triangleq \max_{t\in[0,T]} e^{-\lambda t}\|f\|_\infty$ with $\|f\|_\infty \triangleq \max_{1\leqslant i\leqslant n}|f_i(t)|$ and λ as a positive constant (Arimoto et al., 1984). From this definition, it is clear that for a large λ, the errors at the end of the operation, where t is often large, are much less weighted than those errors at the beginning of the operation. Then, for long trajectories, the tracking error at the end of the operation might rise to an unacceptable value in the sense of the ∞-norm while the λ-norm is still a small value. For this reason, a huge overshoot of error might be observed; this phenomenon is referred to as a bad learning transient (Lee & Bien, 1997; Longman, 2000).

There have been many efforts to generate a good learning transient(Cai, Freeman, Lewin, & Rogers, 2008; Chang, Longman, & Phan, 1992; Chen & Moore, 2001; Hakvoort, Aarts, van Dijk, & Jonker, 2008; Lee & Bien, 1997; Moore, Chen, & Bahl, 2002, 2005; Sadegh, Hu, & James, 2002; Tomizuka, 1987; Tomizuka, Tsao, & Chew, 1989; Wang, 2000; Wang & Ye, 2005; Zhang, Wang, & Ye, 2005; Zhang, Wang, Ye, Wang, & Zhou, 2008). One simple way is to introduce a low-pass filter to cut off high frequency components that can cause the bad learning transient. However, ILC with such a low-pass filter does not have the ability to suppress those error components beyond the filter's cutoff frequency, and zero tracking error cannot be achieved. Therefore, this method introduces a trade-off between tracking accuracy and learning behavior. Another natural way is to tune the learning gain on the iteration axis (Wirkander & Longman, 1999) or on the time axis (Lee & Bien, 1997). The limitation of these learning gain tuning methods is that they require much knowledge of the system, and a very small learning gain can also yield a bad learning transient (Chang et al., 1992). Other methods include the bisection method (Chang et al., 1992) and a scheme with a reduced sampling rate in the first step to deal with initial state error (Hillenbrand & Pandit,

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2000). The difficulty in the former is that it is difficult to choose the number of steps to meet a desired error tolerance restriction (Chang et al., 1992), while the latter only focuses on the initial state error.

Consider that in the ∞-norm sense, an exponential convergence condition for a P-type ILC is derived (Moore, 2001). However, the condition in (Moore, 2001) is often difficult to satisfy. To design a feedback controller to ensure that the condition holds is inconvenient, time-consuming, and induces a high cost. Alternatively, a simple and effective solution is to reduce the sampling rate to force the condition in Moore (2001) to hold. Based on this idea, a pseudo-downsampled ILC (Zhang et al., 2008) is proposed. In this scheme, the downsampled signals are used in learning, which results in loss of information for those in-between sampling points. A two-mode ILC (Zhang, Wang, Ye, Wang, & Zhou, 2007) is proposed to compensate for this loss. In the two-mode ILC, a conventional ILC with the system sampling rate is used in the low frequency range, while a pseudo-downsampled ILC is applied to high frequency components beyond the learnable bandwidth. Although two-mode ILC can compensate for the lost information in the low frequency range, the lost information in the high frequency range cannot be compensated. Therefore, in theory, these two schemes cannot achieve zero tracking error.

In this paper, a new multirate cyclic pseudo-downsampled ILC is proposed to track trajectories with high frequency components. In this scheme, the feedback control system has a sampling rate with a period of T (sampling period of the feedback system), which is referred to as the *feedback sampling rate* hereinafter. ILC has a sampling rate with a period of mT, which is a downsampled slower rate and is referred to as the *ILC sampling rate* hereinafter. The ratio m between the two sampling periods is referred to as the *sampling ratio*. Since all the signals are sampled at the *feedback sampling rate* while ILC merely uses the downsampled signals (realized by software), this downsampling process is termed as pseudo-downsampling. With this downsampling, ILC updating is carried out at every m sampling points and these sampling points are referred to as *downsampling points*. For the next iteration, the *downsampling points* shift forward by a time interval of T. Because of this time shift, downsampling is a cyclic process with a period of m cycles on the iteration axis and therefore, the input to every sampling point at the *feedback sampling rate* is updated once every m cycles. Due to this cyclic input update based on the pseudo-downsampled signals, this ILC scheme is referred to as the cyclic pseudo-downsampled ILC. The benefits of this scheme include the tracking of trajectories with high frequency components, the ability to deal with initial state error, elimination of the need for a filter design, improvement of the tracking accuracy, and the reduction of computation and memory size. Experimental results are presented to verify the proposed method.

The paper is organized as follows. In Section 2, the idea of downsampled learning is briefly introduced, which is followed by design and implementation of the proposed cyclic pseudo-downsampled ILC in Section 3. A series of experiments are presented in Section 4 and concluding remarks are given in Section 5.

2. Downsampled learning

Consider a discrete-time linear single input single output (SISO) system

$$\begin{cases} x_{f,j}(k+1) = A_f x_{f,j}(k) + B_f u_{f,j}(k) + w_{f,j}(k) \\ y_{f,j}(k) = C_f x_{f,j}(k) + v_{f,j}(k) \end{cases} \tag{1}$$

with a one-step-ahead learning update law

$$u_{f,j+1}(k) = u_{f,j}(k) + \Gamma e_{f,j}(k+1) \tag{2}$$

where $k \in [0, p-1]$, p is the number of total sampling points of a given trajectory to be followed, the state $x_{f,j}$ is a n dimensional vector, the input $u_{f,j}$ and the output $y_{f,j}$ are both scalars, the subscript j is the iteration index, f denotes the feedback system sampling rate, and $w_{f,j}$ and $v_{f,j}$ are the repeated state disturbances and output disturbances, respectively. The error is $e_{f,j}(k) = y_d(k) - y_{f,j}(k)$ with y_d as the desired trajectory. Γ is the learning gain.

We define an operator $\delta_{f,j} z(k) = z_{f,j}(k) - z_{f,j-1}(k)$ (Longman, 2000) to obtain the difference value of any variable in two successive iterations. Applying this to the output and assuming the same initial state, i.e., $x_{f,j}(0)$ is the same for all j, gives

$$e_{f,j+1} = Q e_{f,j} \tag{3}$$

where $e_{f,j} = [e_{f,j}(1), e_{f,j}(2), \ldots, e_{f,j}(p)]^T$ and

$$Q = \begin{bmatrix} 1-\Gamma C_f B_f & 0 & \ldots & 0 \\ -\Gamma C_f A_f B v & 1-\Gamma C_f B_f & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\Gamma C_f A_f^{p-1} B_f & -\Gamma C_f A_f^{p-2} B_f & \ldots & 1-\Gamma C_f B_f \end{bmatrix}$$

If all eigenvalues of Q are less than one, then

$$\lim_{j\to\infty} \|e_{f,j}\| \to 0$$

Unfortunately, this condition cannot guarantee a good transient. To obtain a monotonic decay of error along the iteration axis, by taking the ∞-norm on both sides of (3), we arrive at

$$\|e_{f,j+1}\|_\infty \leqslant \|Q\|_\infty \|e_{f,j}\|_\infty \tag{4}$$

where the ∞-norm of a matrix X with entities $x_{i,j}$ is given by $\|X\|_\infty = \max_i \sum_j |x_{i,j}|$.

Hence, the monotonic error decay in the sense of the ∞-norm requires

$$\|Q\|_\infty \leqslant 1 \tag{5}$$

The condition of (5) can be analyzed in two cases:

Case 1: If $(1-\Gamma C_f B_f) > 0$ and $|1-\Gamma C_f B_f| < 1$, (5) holds. The condition for monotonic decay of error in the sense of the ∞-norm can be derived as (Moore, 2001):

$$|C_f B_f| \geqslant \sum_{i=1}^{p-1} |C_f A_f^i B_f| \tag{6}$$

Case 2: If $(1-\Gamma C_f B_f) < 0$ and $|1-\Gamma C_f B_f| < 1$, (5) still holds. In this case, the condition for monotonic decay of error in the sense of ∞-norm becomes (Moore, 2001)

$$|C_f B_f| < \frac{2}{\Gamma} - \sum_{i=1}^{p-1} |C_f A_f^i B_f| \tag{7}$$

The original conditions in Moore (2001) are given in the 1-norm. Since Q is a Toeplitz matrix, these conditions hold for the ∞-norm.

However, condition (6) is related only to the system dynamics. For a discrete-time system with a given sampling rate, its Markov parameters are constants and condition (6) often cannot be satisfied. Although condition (7) has an additional freedom Γ, this condition is also difficult to satisfy if a large Γ is chosen to improve the convergence speed. In addition, a large Γ is prone to violating premises $(1-\Gamma C_f B_f) < 0$ and $|1-\Gamma C_f B_f| < 1$.

Fortunately, a hidden freedom—sampling rate—can be used to make these two conditions easier to satisfy. For a continuous-time system A_c, its zero order hold equivalent with a sampling period of T is (Hillenbrand & Pandit, 2000)

$$A = e^{A_c T}$$