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Steady-state iterative learning control for a class of nonlinear PDE processes☆

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In this paper, a P-type steady-state iterative learning control (ILC) scheme is applied to the boundary control of a class of nonlinear processes described by partial differential equations (PDEs), which cover many important industrial processes such as heat exchangers, industrial chemical reactors, biochemical reactors, and biofilters. Under several practical properties such as physical input–output monotonicity, process stability, and repeatability, the control problem is first transformed to an output regulation problem in the spatial domain. Next, the learning convergence condition of steady-state ILC, the learning rate, as well as the robustness, are derived through rigorous analysis. The adopted ILC scheme fully utilizes the process repetition and deals with both parametric and non-parametric uncertainties. In the end, an illustrative example is presented to demonstrate the performance of the proposed ILC scheme.

1. Introduction

Iterative learning control is a scheme that learns and improves the performance of a system when the control task repeats [1–3]. For processes in repetitive operation mode, e.g., casting [4], rapid thermal processing [5], chemical polymerization/crystallization [6,7], industrial injection molding [8,9], ILC has achieved remarkable control performances. In those applications, different ILC algorithms, from P-type ILC to higher-order PD-type ILC, have been explored and tested. The main idea of ILC is to incorporate control and error information of the previous iterations into the control for the current iteration so as to improve the tracking accuracy, and ultimately achieve the desired control performance. ILC is playing an important role in controlling repeatable processes with parametric or non-parametric uncertainties [10].

Despite the significant progress of ILC for finite dimensional systems, studies on ILC for distributed parameter processes or infinite-dimensional processes are limited due to the interweave of 3D dynamics in the time, space, and iteration domains. Three related works were found in this field, one is based on semi-group theory and the other two are based on Lyapunov theory. In [11], the design of P-type and D-Type ILC laws for a class of infinite-dimensional linear systems is considered using variable separation technique and distributed control. In [12], differential-difference type ILC is argumented with P controller to attenuate the unknown periodic speed variation for a stretched string system on a transporter. In [13], the similar ILC scheme is combined with PD controller to compensate for the unknown periodic motion on the right end for a class of axially moving material systems. While the control variables are boundary forces in [12,13], they are mainly designed for the stability maintainance of mechanical processes. As a matter of fact, many chemical, biochemical, nuclear, thermo, and hydro dynamic processes are inherently nonlinear and are characterized by the presence of strong spatial variations due to the coupling of diffusive and convective mechanisms [14]. The mathematical models, which describe the spatiotemporal behavior of these processes, are typically obtained from the dynamic conservation principles and formulated by partial differential equations. In this work, we extend the framework of ILC to a class of single-input single-output (SISO) quasi-linear PDE processes that include many important industrial processes as special cases, e.g., industrial chemical reactors [15], heat exchangers [16], biochemical reactors [17], and biofilters for air and water pollution control [18]. The control objective is to iteratively tune the velocity boundary condition on one side such that the boundary output at the other side can be regulated to a desired level.

For practical applications of ILC, it is imperative to consider robustness and learning rate when process uncertainties are present [19]. When dealing with distributed parameter systems, especially in the scenario of boundary control and nonlinear PDE processes, the ILC design and property analysis become far more challenging. The existing ILC design and analysis [1–3,20,21] may not be directly applicable. The main result of the paper reveals that, under some physically reasonable properties, such as input–output monotonicity, process stability, and repeatability, the control problem can be first transformed into an output regulation problem in

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the spatial domain. Without considering the transient behavior of process, the learning convergence condition of steady-state ILC, the learning rate, as well as the robustness, can be derived through rigorous analysis.

One motivation of choosing ILC is to avoid using an instantaneous feedback control scheme, because feedback may result in instability problems in the presence of time delay and system uncertainties. On the contrary, ILC is a kind of feedforward control law, while does not cause any closed-loop stability issue in the time domain. Moreover, compared with adaptive control [22,23] and backstepping design [24,25] for the boundary control of PDE processes, the steady-state ILC mechanism is much simple, able to fully utilize the task repetition. Thus, the control scheme is suitable for implementation on biochemical reactors [15] and industrial chemical reactors [17].

Throughout this paper, denote $\mathbf{R}$ the set of real numbers, $\mathbf{R}^+$ the positive part of $\mathbf{R}$, and $\mathcal{H}(E, \mathbf{R}^n)$ an infinite-dimensional Hilbert space of n-dimensional vector functions defined on a domain E. For a scalar function $f(t)$ or a vector-valued function $\mathbf{f}(t)$, $\bar{f}$ and $\bar{\mathbf{f}}$ denote their steady states respectively. Denote $|\cdot|$ an absolute value, $\|\cdot\|$ the vector norm or induced matrix norm, of a quantity.

For clarity, theory proofs are all arranged in Appendices.

2. System description and problem statement

Consider the following SISO quasi-linear PDE process with velocity boundary control under repeatable environment [26]

$$\frac{\partial \mathbf{c}_i(z,t)}{\partial t} = -B\frac{\partial(v_i(z,t)\mathbf{c}_i(z,t))}{\partial z} + D\frac{\partial^2 \mathbf{c}_i(z,t)}{\partial z^2} + \mathbf{f}(\mathbf{c}_i(z,t),z), \tag{1}$$

$$\frac{\partial v_i(z,t)}{\partial t} = -v_i(z,t)\frac{\partial v_i(z,t)}{\partial z}, \tag{2}$$

with the controlled output

$$y_i = h(\mathbf{c}_i(L,t)), \tag{3}$$

the boundary conditions

$$\mathbf{c}_i(0,t) = C_1, \quad A_2\mathbf{c}_i(L,t) + B_2\frac{\partial \mathbf{c}_i(L,t)}{\partial z} = C_2, \tag{4}$$

$$a_1u_i(t) + b_1\frac{\partial v_i(0,t)}{\partial z} = c_1, \quad a_2v_i(L,t) + b_2\frac{\partial v_i(L,t)}{\partial z} = c_2, \tag{5}$$

as well as the initial conditions

$$\mathbf{c}_i(z,0) = \mathbf{c}_0(z), \quad v_i(z,0) = v_0(z). \tag{6}$$

Here, i denotes the iteration number of process, $\mathbf{c}_i \in \mathcal{H}([0,L]\times[0,T],\mathbf{R}^n)$ is the vector of process variables with $0<L$, $T<\infty$, $v_i \in \mathcal{H}([0,L]\times[0,T],\mathbf{R})$ is the fluid velocity. Moreover, $y_i = h(\mathbf{c}_i) \in \mathbf{R}$ is the controlled output, $u_i(t) = v_i(0,t) \in \mathbf{R}$ is the boundary input, $z \in [0,L]$ is the spatial coordinate, and $t \in [0,T]$ is the time. In the system, A_2, B_2, C_1, C_2, $B(>0)$, and $D(>0)$ are matrices of suitable dimension, and $a_j, b_j, c_j, j=1,2$ are a set of scalar parameters. Among all of these parameters, A_2, B_2, C_2, a_j, b_j, c_j, $j=1,2$ are parametric unknowns but the remaining C_1, B, D are assumed to be known in the nominal design and perturbed by some bounded uncertainties in the consequent robust design. In (1) and (3), the two C^1 nonlinear functions $\mathbf{f}(\mathbf{c},z)$ and $h(\mathbf{c})$ involve some uncertainties but satisfy the Lipschitz conditions, namely for $\mathbf{c}_1, \mathbf{c}_2 \in \mathcal{H}([0,L]\times[0,T],\mathbf{R}^n)$, $z\in[0,L]$ there exist a known integrable Lipschitz function $\omega_{\mathbf{f}}(z)$ and another known Lipschitz constant ω_h such that

$$\|\mathbf{f}(\mathbf{c}_1,z) - \mathbf{f}(\mathbf{c}_2,z)\| \le \omega_{\mathbf{f}}(z)\|\mathbf{c}_1 - \mathbf{c}_2\|, \tag{7}$$

and

$$\|h(\mathbf{c}_1) - h(\mathbf{c}_2)\| \le \omega_h\|\mathbf{c}_1 - \mathbf{c}_2\|. \tag{8}$$

The above PDE models generally describe the diffusion-convection phenomena in some open-loop processes. Many important industrial processes can be formulated within this modeling framework. For instance, when regulating the total amount of output flow pollutions (e.g., toluene vapor) via manipulations of the input flow rate in an airstream biofilter, the vector $\mathbf{c}_i(z,t)$ denotes the distribution of pollutants in the i th trial, the parameters B and D are the convection and diffusion coefficients of the pollutions in filter, and the term $\mathbf{f}(\mathbf{c}_i(z,t),z)$ represents the bio-reaction rate that affects the pollution concentration in filter. Meanwhile, the fluid velocity field is approximated in (2) along the process, and it keeps the basic feature for our control design and analysis. When the fluid is incompressible, it can be written as $v_i(z,t) = u_i(t)$ directly. It is worthy of noticing that even if $\mathbf{f}(\mathbf{c}_i(z,t),z)$ is a linear function and the fluid is incompressible, the control problem is still nonlinear due to the product $\partial(v_i\mathbf{c}_i)/\partial z$. Our discussion in Section 3 states that the proposed ILC scheme is also applicable when more sophisticated velocity field models are considered.

For the PDE process (1)–(6), the following restrictions or assumptions are usually satisfied: velocity saturation and output accessibility. The details are as follows:

Assumption 1. The process is operated within the velocity restriction $v(z,t) \in [v_{\min}, v_{\max}]$, with $v_{\max} > v_{\min} \ge 0$.

Assumption 2. The controlled output $y = h(\mathbf{c})$ is available by measurement with some time delay.

Remark 1. From the physical point of view, Assumptions 1 and 2 are practically sound. The output accessibility enables correction by control either temporally or iteratively. The existence of velocity bounds facilitates the convergence analysis by choosing appropriate learning gains. In ILC design, only a rough estimation of velocity bounds is needed.

Let $y^* \in \mathbf{R}$ be a given set-point, which can be achieved at the process output side by tuning the boundary velocity $u_i(t)$. Our primary idea is to design a simple P-type ILC law

$$u_{i+1}(t) = u_i(t) + \rho_i(y^* - y_i(t)), \tag{9}$$

where i is the iteration number and ρ_i is a learning gain, such that the regulation error $y^* - y_i(t)$ converges into an acceptable neighborhood of zero in a pointwise manner within a finite number of iterations. However, looking into the closed-loop system (1)–(6) and (9), dynamics in three different domains (time, space, and iteration) are now involved together. ILC convergence analysis in such situation would be very difficult, if not impossible, and very different from existing results, because most ILC consider systems with pure temporal dynamics or pure spatial dynamics. Although convergence is derived for a class of parabolic distributed parameter systems in [11] by using the contractive mapping method, the ILC scheme presented there belongs to distributed control and deals with linear systems. Our process model is nonlinear and the actuation position is at the boundary.

Considering that the control task is to regulate the output at the other side of boundary to a set-point value, which is obviously time-invariant, it suffices to neglect the transient behavior of system in the time domain and consider the steady-state convergence in the spatial domain only. As such, the control task is transformed to a steady-state ILC problem for a process described by ordinary differential equations. Since ILC is a kind of feedforward control law, it would not cause any closed-loop stability issue in the time domain. Consequently, the convergence of steady-state ILC along the iteration domain implies that the set-point control task is fulfilled strictly.