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Iterative learning approaches to design finite-time consensus protocols for multi-agent systems ☆

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ABSTRACT

In this paper, the finite-time output consensus problem of multi-agent systems is considered by using the iterative learning control (ILC) approach. Two classes of distributed protocols are constructed from the two-dimensional system point of view (with time step and iteration number as independent variables), and are termed as iterative learning protocols. If learning gains are chosen appropriately, then all agents in a directed graph can be enabled to achieve finite-time consensus with the iterative learning protocols. Moreover, all agents in a directed graph can be guaranteed to reach finite-time consensus at any desired terminal output if the iterative learning protocols are improved by introducing the desired terminal output to some (not necessarily all) of the agents. Simulation results are finally presented to illustrate the performance and effectiveness of our iterative learning protocols.

1. Introduction

Cooperative control of multi-agent systems has attracted considerable attention due to its wide applications in many areas such as mobile robots, unmanned air vehicles, satellites, and air-/spacecrafts. One of the basic problems in these applications is the consensus of multi-agent systems [1–5]. Until very recently, there have been numerous new developments devoted to proposing protocols and solutions to consensus problems; see, e.g., [6–17]. Among these studies, an important issue discussed in the protocol design is how to achieve consensus within a finite time [14–17]. It is mainly because the convergence rate is often considered as an important index for the performance evaluation of consensus protocols. As a consequence, the finite-time consensus is usually required in practice. Besides, the finite-time consensus is desirable in many situations such as the case requiring crucial control accuracies, and it can also show better performance in disturbance rejection and robustness against uncertainty [15].

With regard to effective finite-time techniques, iterative learning control (ILC, [18]) has been well known for its ability to achieve a perfect tracking of the desired target. Recently, ILC has been developed to improve the control performances of complex multiple systems, e.g., see [19] for satellites, [20] for multi-agent systems, and [21] for mobile robots. It has been shown that the introduction of ILC can help to provide some desired performances in achieving control objectives of multiple systems such as trajectory-keeping, reference-tracking, and leader–follower formation. Motivated by the developments of [19–21] and the important applications of consensus theory claimed in [14–17], this paper attempts to solve the consensus problems for multi-agent systems through designing some distributed protocols in the two-dimensional system framework of ILC[1]. To our knowledge, there have been no studies on the consensus theory from the viewpoint of two-dimensional systems, and all existing consensus results are proposed in the one-dimensional system framework evolving along the time axis.

In this paper, effective distributed protocols are presented by incorporating the idea of ILC, and thus are termed as iterative learning protocols. On one hand, an iterative learning protocol improves its own performances gradually via an iteration process, which results in fundamentally a two-dimensional system with evolution along two independent axes, like the general ILC [18]. On the other hand, an iterative learning protocol is constructed for each agent based on communications with its neighbors, rather than the fixed targets, like the general protocols [4]. With these two features, it shows that iterative learning protocols can be used to effectively enable all agents to achieve the finite-time consensus, and design conditions can be obtained for the selections of learning gains. If, for any prescribed desired terminal output, it

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[1] Two-dimensional systems represent the class of systems that depend on two independent variables. Hence, each ILC creates a two-dimensional system, with time step and iteration number as two independent variables [18].

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can be detected by some of the agents, then the iterative learning protocols can be modified to solve the finite-time consensus problem just at the desired terminal output, which coincides with the perfect tracking objective of ILC. The main contributions of the proposed results are as follows: (i) new methods to design consensus algorithms are provided by using the iterative learning approach, which can determine a protocol to enable the multi-agent system in a directed graph to achieve consensus in a finite time, and (ii) new methods to analyze iterative learning algorithms are developed by combining the graph theory, which can address the convergence problem of ILC processes not satisfying the contraction mapping principle. To verify the effectiveness of our iterative learning protocols, simulation results are finally given by considering a system of six agents which have relative degrees of different orders.

This paper is organized as follows. In Section 2, some preliminaries are introduced for the graph theory, and two finite-time consensus objectives are formulated under the two-dimensional system framework of ILC. Two classes of iterative learning protocols are presented in Section 3, together with their consensus analysis. Simulation results, and then conclusions, are given in Sections 4 and 5, respectively.

Notations: Let $\mathcal{I}_n = \{1, 2, \ldots, n\}$, $\mathbf{1}_n = [1, 1, \ldots, 1]^T \in \mathbb{R}^n$, I and 0 denote the identity matrix and the null matrix with required dimensions, respectively, and diag$\{\cdot\}$ denote the standard block diagonal matrix whose off-diagonal elements are all zero. For an appropriately dimensioned matrix $A = [a_{ij}]$, let $\lambda_i(A)$ denote any eigenvalue of A, $\rho(A)$ denote the spectral radius of A, $A \geq 0$ denote a nonnegative matrix whose entries are all nonnegative, and $A \circ B = [a_{ij}b_{ij}]$ denote the Hadamard product of two matrices A and B, where $B = [b_{ij}]$. Given any vector $\boldsymbol{a} = [a_1, a_2, \ldots, a_n]^T \in \mathbb{R}^n$, let $\boldsymbol{a} \geq 0$ denote a nonnegative vector whose entries are all nonnegative, $\boldsymbol{a} > 0$ denote a positive vector whose entries are all positive, and $\mathrm{diag}(\boldsymbol{a}) = \mathrm{diag}\{a_1, a_2, \ldots, a_n\}$.

2. Preliminaries and problem formulation

2.1. Preliminaries

Let $\mathcal{G}$ denote an nth order *directed graph* that consists of a vertex set $\mathcal{V}(\mathcal{G})$ and an edge set $\mathcal{E}(\mathcal{G})$, where $\mathcal{V}(\mathcal{G}) = \{v_i : i \in \mathcal{I}_n\}$ and $\mathcal{E}(\mathcal{G}) \subset \{(v_i, v_j) : v_i, v_j \in \mathcal{V}(\mathcal{G})\}$. The set of neighbors of vertex v_i is denoted by $\mathcal{N}_{v_i} = \{v_j : (v_i, v_j) \in \mathcal{E}(\mathcal{G})\}$, while the associated index set is denoted by $\mathcal{N}_i = \{j : v_j \in \mathcal{N}_{v_i}\}$. A *path* in the directed graph $\mathcal{G}$ is a finite sequence $v_{i_1}, v_{i_2}, \ldots, v_{i_j}$ of vertices such that $(v_{i_l}, v_{i_{l+1}}) \in \mathcal{E}(\mathcal{G})$ for $l = 1, 2, \ldots, j-1$. If there exists a special vertex that can be connected to all other vertices through paths, then $\mathcal{G}$ is said to have a *spanning tree*, and this special vertex is called the *root vertex*. To model the information exchange between any two agents, the directed graph $\mathcal{G}$ is associated with a nonnegative weighted adjacency matrix $\mathcal{A} = [a_{ij}]$, where $a_{ij} > 0 \Leftrightarrow (v_i, v_j) \in \mathcal{E}(\mathcal{G})$ and $a_{ij} = 0$ otherwise. Usually, the weighted directed graph is denoted by $\mathcal{G}(\mathcal{A})$, and $a_{ii} = 0$ is assumed for $i \in \mathcal{I}_n$. Accordingly, the *Laplacian matrix* of the directed graph $\mathcal{G}(\mathcal{A})$ is defined as $\mathcal{L}_{\mathcal{A}} = \Delta - \mathcal{A}$, where $\Delta = \mathrm{diag}\{\Delta_{11}, \Delta_{22}, \ldots, \Delta_{nn}\}$ and $\Delta_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$ for all $i \in \mathcal{I}_n$. In addition, $\mathcal{L}_{\mathcal{A}}$ is also called a Laplacian matrix associated with $\mathcal{A}$. For more details of the graph theory, see [22].

Next, the following result is introduced for the relationship of two directed graphs, which is necessary for the design and analysis of our consensus protocols.

Lemma 1. *Consider the directed graph $\mathcal{G}(\mathcal{A})$, and let the matrix $\mathcal{B} = [b_{ij}]$ satisfy*

$$b_{ij} \begin{cases} > 0, & j \in \mathcal{N}_i \\ = 0, & \text{otherwise}, \end{cases} \quad i \in \mathcal{I}_n.$$

Then the directed graph $\mathcal{G}(\mathcal{B} \circ \mathcal{A})$ has a spanning tree if and only if the directed graph $\mathcal{G}(\mathcal{A})$ has a spanning tree. Furthermore, $\mathcal{L}_{\mathcal{B}\circ\mathcal{A}}$ has exactly one zero eigenvalue if and only if $\mathcal{G}(\mathcal{A})$ has a spanning tree.

Proof. See the Appendix. □

The next two lemmas are directly adopted from the literature [23].

Lemma 2. *Let $A \geq 0$. If $A\boldsymbol{x} = \lambda\boldsymbol{x}$ and $\boldsymbol{x} > 0$, then $\lambda = \rho(A)$.*

Lemma 3. *Let λ be an eigenvalue of matrix A, and $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ be two vectors such that:* (i) *$A\boldsymbol{\alpha} = \lambda\boldsymbol{\alpha}$;* (ii) *$A^T\boldsymbol{\beta} = \lambda\boldsymbol{\beta}$;* (iii) *$\boldsymbol{\alpha}^T\boldsymbol{\beta} = 1$;* (iv) *$|\lambda| = \rho(A) > 0$; and* (v) *$\lambda$ is the only eigenvalue of A with modulus $\rho(A)$. Define $M = \boldsymbol{\alpha}\boldsymbol{\beta}^T$, and then*

$$\left(\lambda^{-1}A\right)^m = M + \left(\lambda^{-1}A - M\right)^m \to M \quad \text{as } m \to \infty.$$

2.2. Problem formulation

This paper attempts to introduce the ILC approach to deal with the consensus problem for complex multi-agent systems that consist of n agents. Let the time and iteration indexes be denoted by t and k, respectively. Then, the ith agent is considered with the following input–output dynamics over $t \in [0, T]$ and $k \in \mathbb{Z}_+$:

$$y_{i,k}(t) = g_i(t) + h_i(q)u_{i,k}(t), \quad i \in \mathcal{I}_n, \tag{1}$$

where $y_{i,k}(t) \in \mathbb{R}$ is the scalar output, $u_{i,k}(t) \in \mathbb{R}$ is the scalar control input, $g_i(t) \in \mathbb{R}$ is the zero-input response function, $h_i(q)$ is the transfer operator with the Markov parameters $h_{i,j}$, $j = r_i, r_i+1, \ldots,$ and r_i is the relative degree of system (1). Clearly, $h_i(q)$ can be expressed in the form of

$$h_i(q) = h_{i,r_i}q^{-r_i} + h_{i,r_i+1}q^{-(r_i+1)} + \cdots = \sum_{j=r_i}^{\infty} h_{i,j}q^{-j}.$$

It is worth pointing out that the order of the state space of each agent can be different from each other, and the relative degree of each agent can also be different from each other, i.e., $r_i \neq r_j$ for $i \neq j$. Moreover, the dynamics of (1) are considered to propagate in two independent directions, which are different from the dynamics for the well-studied consensus problem. The main motivation behind (1) is to describe the dynamics for a class of practical multi-agent systems that operate in an iterative (repetitive, periodic, cyclic) manner, such as formation satellites [19] and formation robots [21].

The control objective of this paper is to find the protocol $u_{i,k}(t)$ to solve a finite-time consensus problem such that

$$\lim_{k\to\infty} \left[y_{i,k}(T) - y_{j,k}(T)\right] = 0, \quad i, j \in \mathcal{I}_n. \tag{2}$$

In other words, the consensus objective (2) implies that all agents asymptotically converge to a stable equilibrium y_C at the terminal time T as the iteration number k increases. But, it is worth noting that the final consensus output cannot be determined as desired, which may depend on many factors such as the initial output and initial input. To this end, the consensus objective of (2) is further considered to be

$$\lim_{k\to\infty} y_{i,k}(T) = y_T, \quad i \in \mathcal{I}_n, \tag{3}$$

where y_T is prescribed as the desired terminal output.

The main idea behind the two objectives of (2) and (3) is to transform the finite-time consensus problems for multi-agent systems into the framework of two-dimensional systems, and then solve them via taking advantage of the effective ILC approaches [18]. It can be seen that, if (2) and (3) are solved, a protocol can be finally obtained through iterative learning to enable all agents to achieve finite-time consensus (see Figs. 2–4 in Section 4 for illustration).