An iterative learning approach to formation control of multi-agent systems

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ABSTRACT

In this paper, an efficient framework is proposed to the formation control problem of multiple agents with unknown nonlinear dynamics, by means of the iterative learning approach. In particular, a distributed D-type iterative learning scheme is developed for the multi-agent system with switching topology, whose switching time and sequence are allowed to be varied at different iterations according to the actual trajectories of agents, and a sufficient condition is derived to ensure that the desired formation can be always preserved from the initial starting location to the final one after some iterations. Simulation results are provided to verify the effectiveness of the proposed approach.

1. Introduction

In recent years, cooperative control of multi-agent systems has been widely studied due to the development of advanced theory of complex systems and its broad applications in many fields including biology, physics, robotics, vehicles and control engineering. Problems of consensus, flocking, formation, rendezvous, coverage and swarming are some important research issues, and the common task is to develop distributed schemes or protocols to ensure the realization of complicated global goals. In particular, the formation control problem is to find a coordinated scheme for networks of multiple agents such that they would reach and maintain some desired, possibly time-varying formation or group configuration. Applications of formation control can be found in the automotive and aerospace areas, ranging from assembling structures, exploration of unknown environments, navigation in hostile environments, to cooperative transportation tasks.

In the literature, researchers proposed many approaches for the multi-agent system to achieve final formation from its initial configuration, roughly categorized as leader-following strategy [1,2], behavior-based approach [3,4] and virtual structure method [5–7], to name a few. In the virtual structure approach, the formation is treated as a single rigid body, and then the motion of each agent can be derived from the trajectory of a corresponding point on the structure. In addition, some researchers also applied consensus protocols to the formation control of multiple agents by appropriately choosing information states on which consensus

is reached [8–11], where consensus means that all agents agree upon some certain quantities of interest [12,13]. For example, in [8], variants of a consensus algorithm were used to tackle the formation control problem of second-order multi-agent systems, and the accurate formation was proved to be asymptotically achieved in the case that information flow was unidirectional. Xiao et al. [11] applied the proposed nonlinear consensus protocol to the formation control, including time-invariant formation, time-varying formation and trajectory tracking, respectively. It turned out that all agents could achieve the expected formation after a finite time.

In the related studies on formation control, the dynamics of agents are usually assumed to be linear and known. Actually, almost all the physical plants contain nonlinearities, and it is difficult to obtain the exact system model. Therefore, studies on formation control of multi-agent systems with unknown nonlinear dynamics are of great necessity. More importantly, in the above research papers, one can only ensure the realization of desired formation when time converges to infinity or after a finite time. However, in some practical applications, all agents have to maintain the desired relative positions that determine the formation structure, during the overall process of executing a given repetitive task, such as cooperative transportation of heavy load [14] and satellite formation keeping [15]. To be specific, in the industrial field, when a group of vehicles or mobile robots cooperate to transport a very large and heavy object in a given formation repetitively, they must maintain the desired formation during the whole transportation process, from the prescribed starting location to the destination.

Recently, Ahn and Chen [16] have overcome the above two limitations by applying the iterative learning control (ILC) approach [17–20] to study the formation control problem of multi-

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agent systems with unknown nonlinear dynamics. It was proved that all agents could keep a desired formation from the initial location to the final one after iterative learning. However, only the formation of cyclic structure was considered in their paper, and how to realize a more general formation deserves further investigation. Secondly, the interaction topology among agents was assumed to be fixed in [16], and the ILC-based formation control for switching networks of multiple agents has not been considered yet. Moreover, the desired relative positions or outputs were only set to two directly connected agents in [16], to describe a prescribed formation configuration. Then, the third problem is how to make the formation description independent of the interaction topology, which is more applicable and particularly essential for handling the switching topology case.

Inspired by [16], we employ the ILC approach to deal with the formation control of general configuration for the multi-agent system with unknown nonlinear dynamics and switching interaction topology, and explicitly address the above three problems. Firstly, by using the virtual structure method or introducing a "virtual leader", the formation control of any given structure is transformed into a suitably defined consensus problem. This framework of formation description turns out to be independent of the time-varying interaction topology. Then, in order to exploit the ILC method, the above consensus problem is further converted into a stability control problem over finite time intervals, to which a distributed D-type iterative learning scheme is proposed. To proceed, a convergence analysis of the obtained ILC process is presented to get a sufficient condition in the form of matrix norm, such that all agents can keep the desired formation during the whole finite-time motion or operation process. Finally, simulation results show that under the proposed scheme, a nonlinear multi-agent system can preserve a desired formation during the whole movement process after a finite number of iterations. It deserves pointing out that the ILC approach is not applicable to the multi-agent coordination system that cannot operate repetitively.

Notations: $\mathbf{1}$ and $\mathbf{0}$ denote the column vectors of appropriate dimensions whose elements are all ones and all zeros, respectively; I_n and 0_n ($0_{n \times m}$) denote the $n \times n$ identity matrix and the $n \times n$ ($n \times m$) zero matrix, respectively; notation D^+ stands for the right derivative operator of a function matrix.

2. Problem reformulation and transformation

2.1. Problem statement

Consider a multi-agent system consisting of n agents with the jth one governed by the following nonlinear dynamics

$$\dot{x}_j(t) = f(x_j(t)) + B(t)u_j(t), \quad j = 1, \ldots, n, \tag{1}$$

where $x_j(t) \in \mathbb{R}^m$ is the state of agent j at time instant t, $u_j(t) \in \mathbb{R}^{m_1}$ is the control input or scheme, and $f(x_j(t))$ is an unknown nonlinear Lipschitz continuous functional in terms of $x_j(t)$. The objective is to design a distributed scheme for the multi-agent system (1) such that the accurate formation can be always kept among agents during the overall motion process.

For the simple cyclic formation addressed in [16], formation errors can be easily defined by the difference values between the real-time relative outputs and the desired ones of any two connected agents. Then, the purpose is to design schemes to drive the above formation errors converge to zero. However, for a general formation configuration whose local connection is tight, irregular and even time-varying, as considered here, the formation error defined in [16] is inapplicable. In view of this, a desired formation is described by the relative positions of all agents to a common non-physical virtual leader. To be specific, we say that the

multi-agent system (1) achieves the desired formation if and only if

$$x_j(t) - d_j(t) = x_l(t) - d_l(t),$$
$$\forall j, l \in \{1, \ldots, n\} \triangleq \mathcal{N}, \ t \in [0, T], \tag{2}$$

where $d_j(t)$ represents the desired relative state of agent j to the common virtual leader, 0 and T are the initial and the stop times, respectively. From (2), it is obvious that the desired relative state from the jth to the lth agents is $d_l(t) - d_j(t)$. Here, we suppose that the multi-agent system operates repetitively to achieve the desired formation over a fixed time interval, motivated by its practical applications presented in the introduction. For simplicity of analysis, it is henceforth assumed that there is no initial formation error, i.e., the initial state condition satisfies (2).

Remark 1. Model (1) covers the first- and second-order linear dynamic systems studied in [2,8] by letting $x_j(t) = r_j(t)$ and $x_j(t) = [r_j^T(t) \ v_j^T(t)]^T$, respectively, where $r_j(t)$ and $v_j(t)$ are the position and the velocity of agent j. For the second-order multi-agent system, the desired relative state of agent j is taken as $d_j(t) = [d_{r_j}^T(t) \mathbf{0}^T]^T$ with $d_{r_j}(t)$ being its desired relative position to the virtual leader, which implies that the group eventually moves with a common velocity and meanwhile preserves the accurate formation.

Directed graphs in algebraic graph theory are applied to model the interaction topologies among agents. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a weighted directed graph of order n with the set of nodes $\mathcal{V} = \{v_1, \ldots, v_n\}$, the set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $A = [a_{jl}]$ with nonnegative adjacency weights a_{jl}. In graph $\mathcal{G}$, the edge (v_l, v_j), from v_l to v_j, represents that information is transmitted from agents l to j. It is stipulated the nonnegative adjacency weight associated with (v_l, v_j) is a_{jl}, which is positive if and only if $(v_l, v_j) \in \mathcal{E}$. Then the set of neighbors of agent j is denoted by $N_j = \{v_l \in \mathcal{V} : (v_l, v_j) \in \mathcal{E}\}$. The Laplacian of a weighted graph $\mathcal{G}$ is defined as $L = D - A$, where $D = \text{diag}\{d_1, \ldots, d_n\}$ with $d_j = \sum_{l=1}^{n} a_{jl}$. To describe the variable topologies, a piecewise-constant switching signal function $\sigma(t) : [0, \infty) \mapsto \{1, \ldots, M\} \triangleq \mathcal{M}$ is defined, where $M \in \mathbb{Z}^+$ denotes the total number of possible interaction graphs. Note that $\sigma(t)$ is a right-hand continuous function. In this paper, each switching graph is assumed to have a spanning tree.

2.2. Problem reformulation

In order to use the ILC theory, we have to transform the formation control problem into an asymptotical stability control problem over a finite time interval. To achieve this, we first reformulate the formation control problem as a consensus problem.

Define the state error of agent j as

$$e_j(t) = x_j(t) - d_j(t), \quad j = 1, \ldots, n. \tag{3}$$

Then the desired formation (2) can be rewritten as

$$e_j(t) = e_l(t), \quad \forall j, l \in \mathcal{N}, \ t \in [0, T]. \tag{4}$$

Thus, the accurate formation is guaranteed by the consensus of all agents on their state errors, defined as in (3). In this way, the formation control problem is reformulated as a consensus problem. Denote $e(t) = [e_1^T(t) \cdots e_n^T(t)]^T \in \mathbb{R}^{nm}$.

Now, the task is to realize the consensus of all agents on their state errors $e_j(t)$ ($j \in \mathcal{N}$), that is, find ways such that $e(t)$ converges to $\mathbf{1} \otimes e_0(t)$, where $e_0(t) \in \mathbb{R}^m$ is the agreement state error. Unfortunately, the ILC method cannot be directly used to solve the above consensus problem, since it usually aims at the monotone convergence analysis of tracking error to zero based on the contraction-mapping theory. Therefore, in order to utilize the ILC theory, we must convert the nonzero equilibrium $\mathbf{1} \otimes e_0(t)$ to the origin, that is, transform the consensus problem into an asymptotical stability problem of another system. To this end, we adopt the tree-type transformation proposed in [13].